



The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
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2023 Canadian Team Mathematics Contest

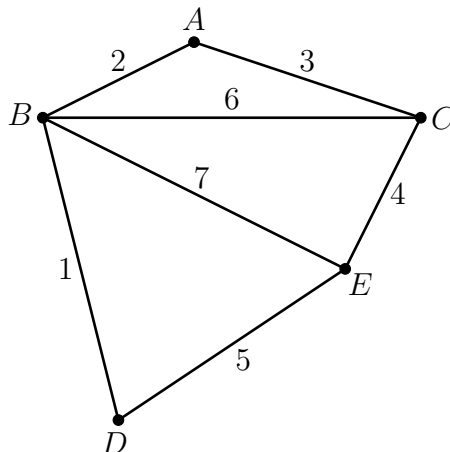
Team Problems (45 minutes)

IMPORTANT NOTES:

- Calculating devices are not permitted.
- Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.

PROBLEMS:

1. A fish and chips truck sells three kinds of fish: cod, haddock, and halibut. During a music festival, the truck sold 220 pieces of fish, of which 40% were haddock and 40% were halibut. How many pieces of cod did the truck sell?
2. If $\frac{x}{2} - 5 = 9$, what is the value of $\sqrt{7x}$?
3. Point A has coordinates $(-13, -23)$ and point B has coordinates $(-33, -43)$. What is the slope of a line that is perpendicular to the line segment AB ?
4. What is the integer equal to $\sqrt{\frac{119^2 - 17^2}{119 - 17} - 10^2}$?
5. If $p + q + r = 70$, $p = 2q$, and $q = 3r$, what is the value of p ?
6. How many ordered triples (a, b, c) of integers satisfy $1 \leq a < b < c \leq 10$ and $b - a = c - b$?
7. The distinct prime factors of 18 are 2 and 3. What is the sum of all of the distinct prime factors of 4446?
8. Using the diagram below, a seven digit integer can be created as follows: trace a path that uses each line segment exactly once and use the labels on the line segments as digits. For example, the path that goes from C to A to B to C to E to B to D and finally to E gives the positive integer 3264715. What is the largest possible positive integer that can be created in this way?

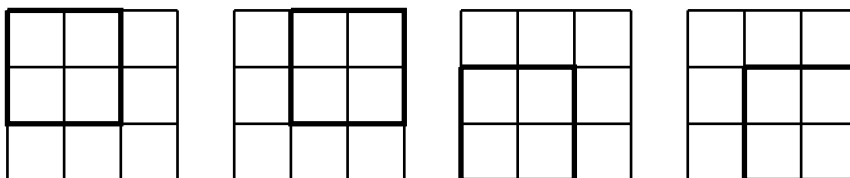


9. Alheri has three hoses. Water flows out of each hose at the same constant rate. Using all three hoses, she starts to fill a swimming pool at 6:00 a.m. and calculates that the pool will be full at exactly 6:00 p.m. on the same day. At 11:00 a.m., one of the hoses unexpectedly stops working. Assuming water still flows out of the other two hoses at the same rate as before, at what time will the pool be full?
10. The lines with the two equations below intersect at the point $(2, -3)$.

$$\begin{aligned}(a^2 + 1)x - 2by &= 4 \\ (1 - a)x + by &= 9\end{aligned}$$

What are the possible ordered pairs (a, b) ?

11. What is the integer equal to $2023^4 - (2022)(2024)(1 + 2023^2)$?
12. For how many integers x is the expression $\frac{\sqrt{75 - x}}{\sqrt{x - 25}}$ equal to an integer?
13. A positive integer is called *mystical* if it has at least two digits and every pair of two consecutive digits, read from left to right, forms a perfect square. For example, 364 is a mystical integer because 36 and 64 are both perfect squares, but 325 is not mystical because 32 is not a perfect square. What is the largest mystical integer?
14. In a 3×3 grid, there are four 2×2 subgrids, each of which is bordered by a thick line in one of the four grids below.



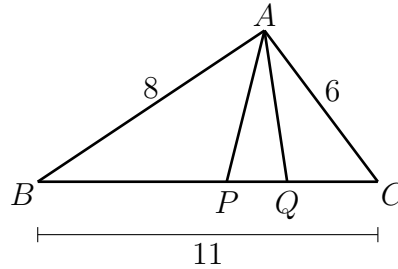
Kasun wants to place an integer from 1 to 4 inclusive in each cell of a 3×3 grid so that every 2×2 subgrid contains each integer exactly once. For example, the grid below and on the left satisfies the condition, but the grid below and on the right does not. In how many ways can Kasun place integers in the grid so that they satisfy the condition?

1	2	3
3	4	1
1	2	3

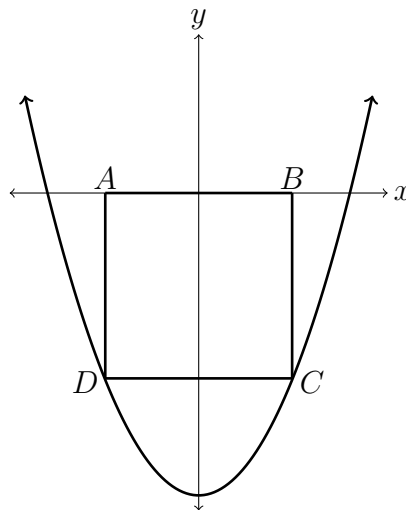
1	3	2
2	4	1
1	3	3

15. There are exactly three real numbers x for which $\left(x - \frac{5}{x}\right)$ is the reciprocal of $(x - 4)$. What is the sum of these three real numbers?

16. In $\triangle ABC$, $AB = 8$, $BC = 11$, and $AC = 6$. The points P and Q are on BC such that $\triangle PBA$ and $\triangle QAC$ are each similar to $\triangle ABC$. What is the length of PQ ?

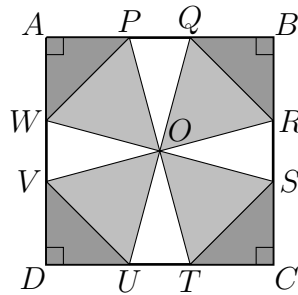


17. Square $ABCD$ has A and B on the x -axis and C and D below the x -axis on the parabola with equation $y = x^2 - 4$. What is the area of $ABCD$?



18. Let $a = \log_4 9$ and $b = 108 \log_3 8$. What is the integer equal to \sqrt{ab} ?
19. Jolene and Tia are playing a two-player game at a carnival. In one bin, there are five red balls numbered 5, 10, 15, 20 and 25. In another bin, there are 25 green balls numbered 1 through 25. In the first stage of the game, Jolene chooses one of the red balls at random. Next, the carnival worker removes the green ball with the same number as the ball Jolene chose. Tia then chooses one of the 24 remaining green balls at random.
- Jolene and Tia win if the number on the ball chosen by Tia is a multiple of 3. What is the probability that they will win?
20. The positive integer d has the property that each of 468, 636, and 867 has the same remainder, r , when divided by d . What is the largest possible value of $d + r$?

21. Square $ABCD$ has centre O . Points P and Q are on AB , R and S are on BC , T and U are on CD , and V and W are on AD , as shown, so that $\triangle APW$, $\triangle BRQ$, $\triangle CTS$, and $\triangle DVU$ are isosceles and $\triangle POW$, $\triangle ROQ$, $\triangle TOS$, and $\triangle VOU$ are equilateral. What is the ratio of the area of $\triangle PQO$ to that of $\triangle BRQ$?



22. The sequence a_1, a_2, a_3, \dots is an arithmetic sequence with common difference 3 and $a_1 = 1$. The sequence b_1, b_2, b_3, \dots is an arithmetic sequence with common difference 10 and $b_1 = 2$. What is the smallest integer larger than 2023 that appears in both sequences?
23. In $\triangle ABC$, $AB = 10$ and $\sin 3A + 3 \sin C = 4$. What is the length of the altitude from C to AB ?
24. The real numbers x , y , and z satisfy both of the equations below.

$$\begin{aligned}x + y + z &= 2 \\xy + yz + xz &= 0\end{aligned}$$

Let a be the minimum possible value of z and b be the maximum possible value of z . What is the value of $b - a$?

25. Let $f(x)$ be a function with the property that $f(x) + f\left(\frac{x-1}{3x-2}\right) = x$ for all real numbers x other than $\frac{2}{3}$. What is the sum $f(0) + f(1) + f(2)$?