



The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Galois Contest

(Grade 10)

Tuesday, April 12, 2022
(in North America and South America)

Wednesday, April 13, 2022
(outside of North America and South America)



UNIVERSITY OF
WATERLOO

Time: 75 minutes

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Do not open this booklet until instructed to do so.

Number of questions: 4

Each question is worth 10 marks

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by



- worth 2 or 3 marks each
- full marks given for a correct answer which is placed in the box
- **part marks awarded only if relevant work** is shown in the space provided

2. **FULL SOLUTION** parts indicated by



- worth the remainder of the 10 marks for the question
- **must be written in the appropriate location** in the answer booklet
- marks awarded for completeness, clarity, and style of presentation
- a correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

- Extra paper for your finished solutions must be supplied by your supervising teacher and inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

NOTE:

1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.
3. For questions marked , place your answer in the appropriate box in the answer booklet and **show your work**.
4. For questions marked , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.
6. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions, and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x -intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
7. No student may write more than one of the Fryer, Galois and Hypatia Contests in the same year.

1. Alice and Bello contributed to the cost of starting a new business. The ratio of Alice's contribution to Bello's contribution was 3 : 8.



(a) If the cost of starting the new business was \$9240, what was Bello's contribution to this starting cost?



(b) Alice and Bello divided up all profits in the first year of the business in the same ratio, 3 : 8. Alice's share of the first year's total profit was \$1881. What was the total profit of the business for the first year?



(c) In the second year, the business was changed so the share of that year's profits for Alice and Bello was in the ratio of 3 : (8 + x). If the profit for the second year was \$6400 and Bello's share of that profit was \$5440, determine the value of x .

2. In the diagram shown, line L_1 has equation $y = \frac{3}{2}x + k$, where $k > 0$, and L_1 intersects the y -axis at P . A second line, L_2 , is drawn through P perpendicular to L_1 , and intersects the x -axis at Q . A third line, L_3 , is drawn through Q parallel to L_1 , and intersects the y -axis at R .



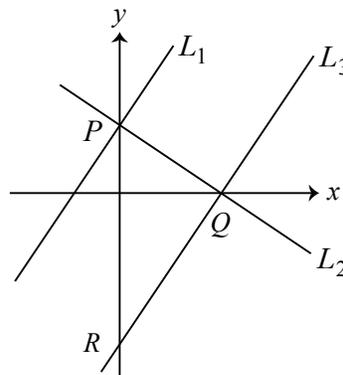
(a) What is the slope of L_2 ?



(b) Written in terms of k , what is the x -coordinate of point Q ?



(c) If the area of $\triangle PQR$ is 351, determine the value of k .



3. The *prime factorization* of 324 is $2 \times 2 \times 3 \times 3 \times 3 \times 3$ or $2^2 \times 3^4$. Notice that 324 is a perfect square because it can be written in the form $(2 \times 3^2) \times (2 \times 3^2)$. The prime factorization of 63 is $3^2 \times 7$. Notice that 63 is not a perfect square, but 63×7 is a perfect square, because $63 \times 7 = 3^2 \times 7^2 = (3 \times 7) \times (3 \times 7)$.



(a) The product $84 \times k$ is a perfect square. If k is a positive integer, what is the smallest possible value of k ?



(b) The product $572 \times \ell$ is a perfect square. If ℓ is a positive integer less than 6000, what is the greatest possible value of ℓ ?



(c) Show that if m is a positive integer less than 200, then $525\,000 \times m$ cannot be a perfect square.



(d) The list $10, 10^3, 10^5, \dots, 10^{99}$ contains the fifty powers of 10 with odd integer exponents from 10^1 to 10^{99} , inclusive. Show that the sum of every choice of three different powers of 10 from this list is not a perfect square.

4. A *Bauman string* is a string of letters that satisfies the following two conditions.

- Each letter in the string is $A, B, C, D,$ or E .
- No two adjacent letters in the string are the same.

For example, $AECD$ and $BDCEC$ are Bauman strings of length 4 and length 5, respectively, and $ABBC$ and $DAEEE$ are not Bauman strings.



(a) How many Bauman strings of length 5 are there in which the first letter and the last letter are both A ?



(b) Determine the number of Bauman strings of length 6 that contain more than one B .



(c) Determine the number of Bauman strings of length 10 in which the first letter is C and the last letter is D .



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Thank you for writing the 2022 Galois Contest! Each year, more than 260 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Canadian Intermediate Mathematics Contest or the Canadian Senior Mathematics Contest, which will be written in November 2022.

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- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

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- Obtain information about our 2022/2023 contests
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