Gauss Contest
Grade 8
(The Grade 7 Contest is on the reverse side)

Wednesday, May 12, 2021
(in North America and South America)

Thursday, May 13, 2021
(outside of North America and South America)

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your answer sheet. If you are not sure, ask your teacher to explain it.
4. This is a multiple-choice test. Each question is followed by five possible answers marked A, B, C, D, and E. Only one of these is correct. When you have made your choice, enter the appropriate letter for that question on your answer sheet.
5. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C. There is no penalty for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
6. Diagrams are not drawn to scale. They are intended as aids only.
7. When your supervisor instructs you to start, you will have sixty minutes of working time.

The name, school and location of some top-scoring students will be published on the website, cemc.uwaterloo.ca. On this website, you will also be able to find copies of past Contests and excellent resources for enrichment, problem solving and contest preparation.
Scoring: There is no penalty for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $999 + 999$ is
   (A) 2999  (B) 181818  (C) 1998  (D) 999999  (E) 198

2. The perimeter of an equilateral triangle is 15 m. What is the length of each side of the triangle?
   (A) 7.5 m  (B) 5 m  (C) 3.75 m  (D) 10 m  (E) 17 m

3. What is the greatest multiple of 4 that is less than 100?
   (A) 99  (B) 96  (C) 97  (D) 98  (E) 94

4. In the graph shown, which of the following statements is true about the coordinates of the point $P(x, y)$?
   (A) The values of both $x$ and $y$ are positive.
   (B) The value of $x$ is positive and the value of $y$ is negative.
   (C) The value of $x$ is negative and the value of $y$ is positive.
   (D) The values of both $x$ and $y$ are negative.
   (E) The value of $x$ is 0 and the value of $y$ is negative.

5. If $x = -6$, which of the following is greatest in value?
   (A) $2 + x$  (B) $2 - x$  (C) $x - 1$  (D) $x$  (E) $x ÷ 2$

6. A water fountain flows at a steady rate of 500 mL every 6 seconds. At this rate, how long will it take to fill a 250 mL bottle?
   (A) 2 s  (B) 9 s  (C) 3 s  (D) 6 s  (E) 1 s

7. The number 17 is an example of a prime number that remains prime when you reverse its digits (that is, 71 is also prime). Which of the following prime numbers also has this property?
   (A) 29  (B) 53  (C) 23  (D) 13  (E) 41

8. Initially, there are 5 red beans and 9 black beans in a bag. Then, 3 red beans and 3 black beans are added to the bag. If one bean is randomly chosen from the bag, what is the probability that this bean is red?
   (A) $\frac{3}{5}$  (B) $\frac{2}{5}$  (C) $\frac{4}{5}$  (D) $\frac{5}{8}$  (E) $\frac{8}{17}$

9. An ant begins its path at $A$, travels only right or down, and remains on the line segments shown. The number of different paths from $A$ to $C$ that pass through $B$ is
   (A) 2  (B) 3  (C) 4  (D) 5  (E) 6

10. The digits of 2021 can be rearranged to form other four-digit whole numbers between 1000 and 3000. What is the largest possible difference between two such four-digit whole numbers?
    (A) 1188  (B) 1098  (C) 1080  (D) 2088  (E) 999
Part B: Each correct answer is worth 6.

11. In the diagram, \( PQ \) and \( RS \) intersect at \( T \). If \( \angle STQ = 140^\circ \) and \( \angle PTU = 90^\circ \), what is the measure of \( \angle RTU \)?

(A) 30°  (B) 90°  (C) 50°
(D) 40°  (E) 140°

12. Which of the following is the sum of three consecutive integers?

(A) 17  (B) 11  (C) 25  (D) 21  (E) 8

13. Which of the following circle graphs best represents the information in the bar graph shown?

(A) [Diagram A]
(B) [Diagram B]
(C) [Diagram C]
(D) [Diagram D]
(E) [Diagram E]

14. A whole number has exactly 6 positive factors. One of its factors is 16. Which of the following could this number be?

(A) 16  (B) 32  (C) 6  (D) 49  (E) 48

15. The measures of a triangle’s three interior angles are in the ratio 1 : 4 : 7. What are the measures of the angles?

(A) 12°, 48°, 120°  (B) 10°, 40°, 70°  (C) 20°, 25°, 155°
(D) 15°, 60°, 105°  (E) 14°, 56°, 110°

16. The seven numbers 1, 2, 5, 10, 25, 50, 100 repeat to form the following pattern

1, 2, 5, 10, 25, 50, 100, 1, 2, 5, 10, 25, 50, 100, . . .

What is the sum of the 18th and the 75th numbers in the pattern?

(A) 110  (B) 11  (C) 27  (D) 7  (E) 35

17. Gaussville’s soccer team won 40% of their first 40 games. They went on to win \( n \) games in a row. At this point, they had won 50% of the total games they had played. What is the value of \( n \)?

(A) 4  (B) 10  (C) 12  (D) 8  (E) 9

18. In the diagram, the radius of the larger circle is 3 times the radius of the smaller circle. What fraction of the area of the larger circle is not shaded?

(A) \( \frac{5}{6} \)  (B) \( \frac{2}{3} \)  (C) \( \frac{5}{6} \)
(D) \( \frac{7}{5} \)  (E) \( \frac{1}{3} \)
19. Asima and Nile each think of an integer greater than 0. Each of them performs the following operations on their integer: they double it, then subtract 10, and then multiply it by 4. The sum of their results is 440. How many possibilities are there for Asima’s original integer?
(A) 64  (B) 44  (C) 65  (D) 45  (E) 66

20. Ruby and Sam each roll a fair 6-sided die with the numbers 1, 2, 3, 4, 5, and 6 on its faces. Sam subtracts the number on his roll from the number on Ruby’s roll. What is the probability that the result is a negative number?
(A) $\frac{5}{18}$  (B) $\frac{5}{12}$  (C) $\frac{7}{12}$  (D) $\frac{1}{2}$  (E) $\frac{5}{6}$

Part C: Each correct answer is worth 8.

21. When evaluated, the sum of the digits of the integer equal to $10^{2021} - 2021$ is
(A) 18 194  (B) 18 176  (C) 18 167  (D) 18 153  (E) 18 185

22. The prime numbers 23 and 29 are consecutive prime numbers since 29 is the smallest prime number that is greater than the prime number 23. How many positive integers less than 900 can be written as a product of two or more consecutive prime numbers?
(A) 14  (B) 13  (C) 11  (D) 12  (E) 15

23. A dog’s leash is 4 m long and is attached to the corner of a 2 m × 2 m square doghouse at C, as shown. The dog is attached to the other end of the leash, at D. What is the area outside of the doghouse in which the dog can play?
(A) 14π m²  (B) 16π m²  (C) 20π m²  (D) 15π m²  (E) 24π m²

24. Jonas builds a large $n \times n \times n$ cube using $1 \times 1 \times 1$ cubes each having the net shown. What is the smallest value of $n$ for which the sum of the exterior faces of the $n \times n \times n$ cube can be greater than 1500?
(A) 9  (B) 11  (C) 12  (D) 13  (E) 16

25. Square $PQRS$ has sides of length 8. It is split into four rectangular regions by two line segments, one parallel to $PQ$ and another parallel to $QR$. There are $N$ ways in which these lines can be drawn so that the area of each of the four rectangular regions is a positive integer. What is the remainder when $N^2$ is divided by 100?
(A) 9  (B) 61  (C) 1  (D) 41  (E) 36