



The CENTRE for EDUCATION  
in MATHEMATICS and COMPUTING  
*cemc.uwaterloo.ca*

# Canadian Intermediate Mathematics Contest

Wednesday, November 17, 2021  
(in North America and South America)

Thursday, November 18, 2021  
(outside of North America and South America)



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**Time:** 2 hours

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Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Do not open this booklet until instructed to do so.

There are two parts to this paper. The questions in each part are arranged roughly in order of increasing difficulty. The early problems in Part B are likely easier than the later problems in Part A.

## **PART A**

1. This part consists of six questions, each worth 5 marks.
2. **Enter the answer in the appropriate box in the answer booklet.**  
For these questions, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded **only if relevant work** is shown in the space provided in the answer booklet.

## **PART B**

1. This part consists of three questions, each worth 10 marks.
2. **Finished solutions must be written in the appropriate location in the answer booklet.** Rough work should be done separately. If you require extra pages for your finished solutions, they will be supplied by your supervising teacher. Insert these pages into your answer booklet. Write your name, school name, and question number on any inserted pages.
3. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

**At the completion of the contest, insert your student information form inside your answer booklet.**

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*Do not discuss the problems or solutions from this contest online for the next 48 hours.*

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*The name, grade, school and location, and score range of some top-scoring students will be published on the website, [cemc.uwaterloo.ca](http://cemc.uwaterloo.ca). In addition, the name, grade, school and location, and score of some students may be shared with other mathematical organizations for other recognition opportunities.*

## Canadian Intermediate Mathematics Contest

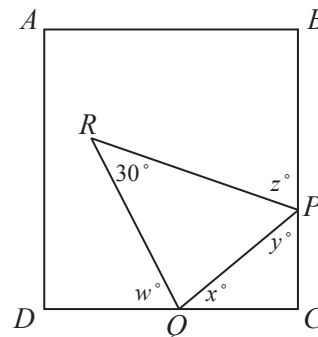
### NOTE:

1. Please read the instructions on the front cover of this booklet.
2. Write solutions in the answer booklet provided.
3. Express answers as simplified exact numbers except where otherwise indicated. For example,  $\pi + 1$  and  $1 - \sqrt{2}$  are simplified exact numbers.
4. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions and specific marks may be allocated for these steps. For example, while your calculator might be able to find the  $x$ -intercepts of the graph of an equation like  $y = x^3 - x$ , you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
5. Diagrams are not drawn to scale. They are intended as aids only.
6. No student may write both the Canadian Senior Mathematics Contest and the Canadian Intermediate Mathematics Contest in the same year.

### PART A

For each question in Part A, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded only if relevant work is shown in the space provided in the answer booklet.

1. Rectangle A has length 6 cm and area  $36 \text{ cm}^2$ .  
Rectangle B has length 12 cm and area  $36 \text{ cm}^2$ .  
Rectangle C has length 9 cm and area  $36 \text{ cm}^2$ .  
The rectangle with the smallest width has a width of  $x$  cm. What is the value of  $x$ ?
2. What is the sum of all of the prime numbers that are greater than 10 and less than 20? (A *prime number* is a positive integer greater than 1 whose only positive divisors are 1 and itself. For example, 7 is a prime number.)
3. Starting on the 22nd floor of their apartment building, Taya goes up the stairs and Jenna goes up by elevator. Beginning when Taya starts going up, Jenna waits for 2 minutes for the elevator. Taya goes up from each floor to the next floor in 15 seconds. The elevator goes up from each floor to the next floor in 3 seconds. Taya and Jenna arrive on the  $n$ th floor at exactly the same time. What is the value of  $n$ ?
4. In the diagram,  $ABCD$  is a rectangle,  $P$  is on  $BC$ ,  $Q$  is on  $CD$ , and  $R$  is inside  $ABCD$ . Also,  $\angle PRQ = 30^\circ$ ,  $\angle RQD = w^\circ$ ,  $\angle PQC = x^\circ$ ,  $\angle CPQ = y^\circ$ , and  $\angle BPR = z^\circ$ . What is the value of  $w + x + y + z$ ?



5. A list of numbers is created using the following rules:
- The first number is 3 and the second number is 4.
  - Each number after the second is the result of adding 1 to the previous number and then dividing by the number before that. In other words, for any three consecutive numbers in the list,  $a, b, c$ , we have  $c = \frac{b+1}{a}$ .
- The rules tell us that the third number in the list is  $\frac{4+1}{3}$  which equals  $\frac{5}{3}$ .
- What is the smallest positive integer  $N$  for which the sum of the first  $N$  numbers in the list is equal to an odd integer that is greater than 2021?
6. Dragomir has 6 pairs of socks in a drawer. Each pair of socks is different from every other pair in the drawer. He randomly removes 4 individual socks from the drawer, one at a time. What is the probability that there is exactly 1 matching pair of socks among these 4 socks?

## PART B

For each question in Part B, your solution must be well-organized and contain words of explanation or justification. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

*Useful Fact for Part B:*

For all real numbers  $a$  and  $b$ ,  $(a + b)^2 = a^2 + 2ab + b^2$ .

1. Karol uses Gluze and Blurpos to make Zippies. His recipe uses 4 cups of Gluze and 3 cups of Blurpos to make 60 Zippies. He can also make larger batches of Zippies using these same ratios.
- (a) On Monday, Karol uses 28 cups of Gluze when he makes Zippies. Determine the number of cups of Blurpos that he uses.
  - (b) On Tuesday, Karol uses 48 cups of one of the ingredients and  $N$  cups of the other ingredient when he makes Zippies. Determine the two possible values of  $N$ .
  - (c) On Wednesday, Karol has 64 cups of Gluze and 42 cups of Blurpos available. Determine the largest number of Zippies that he can make.
  - (d) On Thursday, Karol makes Zippies to sell. His only costs come from purchasing Gluze and Blurpos. He sells each Zippie for \$0.50 and makes a profit of \$0.30 from each Zippie sold. Karol pays \$1.80 for each cup of Gluze. Determine how much Karol pays per cup of Blurpos.

2. (a) In Figure 1, trapezoid  $ABED$  is formed by joining  $\triangle BCE$  (which is right-angled at  $C$ ) to rectangle  $ABCD$ . If  $AB = 3$ ,  $CE = 6$  and the area of trapezoid  $ABED$  is 48, determine the length of  $BE$ .
- (b) In Figure 2, circles with centres  $P$  and  $Q$  touch a line  $\ell$  (that is, are tangent to  $\ell$ ) at points  $S$  and  $T$ , respectively, and touch each other at point  $X$ . Because the circles touch at  $X$ , line segment  $PQ$  passes through  $X$  and so  $PQ = PX + XQ$ . Because the circles touch  $\ell$  at  $S$  and  $T$ ,  $PS$  and  $QT$  are perpendicular to  $\ell$ . If the circle with centre  $P$  has a radius of 25 and the circle with centre  $Q$  has a radius of 16, determine the area of trapezoid  $PQTS$ .
- (c) Figure 3 is formed by adding a circle of radius  $r$  to Figure 2 that touches  $\ell$  and each of the two other circles, as shown. Determine the value of  $r$ .

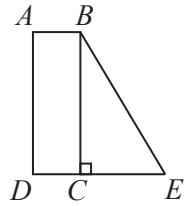


Figure 1

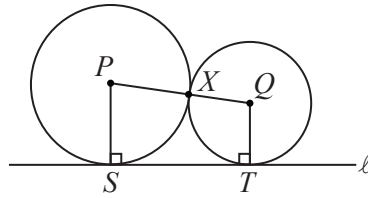


Figure 2

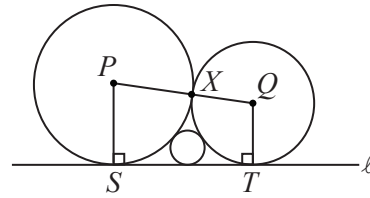


Figure 3

3. Suppose that  $N$  is some positive integer. Alphonse and Beryl play a game in which they take turns adding integers to a running total. Alphonse always goes first and the beginning total is always 0. On turn  $k$ , the player whose turn it is may add to the running total any one of the integers from 1 to  $k$ , inclusive. The winner is the player who brings the total to exactly  $N$ .

For example, if  $N = 10$ , a game *may* proceed as shown below. (In this example, Alphonse wins, although he might not always win when  $N = 10$ .)

Turn #	Player	Possible integers	Choice	Running total
1	Alphonse	1	1	1
2	Beryl	1, 2	2	3
3	Alphonse	1, 2, 3	1	4
4	Beryl	1, 2, 3, 4	3	7
5	Alphonse	1, 2, 3, 4, 5	3	10

For each value of  $N$ , one player has a winning strategy. (A *winning strategy* is a method which that player can use to win no matter the choices of the other player.)

- If  $N = 8$ , show that Beryl has a winning strategy by describing a specific method and explaining why this method guarantees that Beryl will win.
- If  $N = 17$ , determine which of the two players has a winning strategy. Your solution should describe a specific strategy and explain why it is a winning strategy.
- Determine the smallest positive integer  $m$  with  $m > 2021$  for which Alphonse has a winning strategy when  $N = m$  and Beryl has a winning strategy if  $N = m + 1$ .

2021  
Canadian  
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(English)