The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

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2020 Canadian Team Mathematics Contest

Individual Problems

IMPORTANT NOTES:

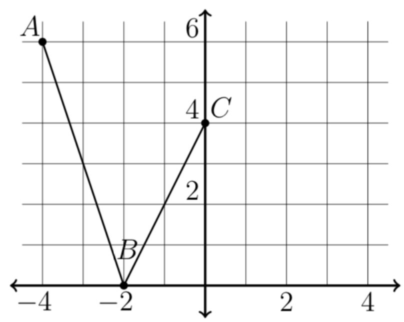
- Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) previously stored information such as formulas, programs, notes, etc., (iv) a computer algebra system, (v) dynamic geometry software.
- Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.

PROBLEMS:

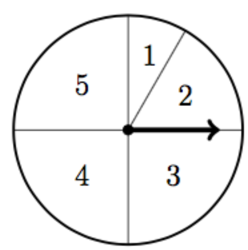
1. What is the value of $\dfrac{24+12}{4^2-4}$?

2. If $3k = 10$, what is the value of $\dfrac{6}{5}k - 2$?

3. Segment AB is reflected in the y-axis to obtain ED. Segment BC is reflected in the y-axis to obtain DC. Determine the sum of the slopes of DC and ED.



4. A spinner was created by drawing five radii from the centre of a circle. The first four radii divide the circle into four equal wedges. The fifth radius divides one of the wedges into two parts, one having twice the area of the other. The five wedges are labelled as pictured with the wedge labeled by 2 having twice the area of the wedge labeled by 1. Determine the probability of spinning an odd number.



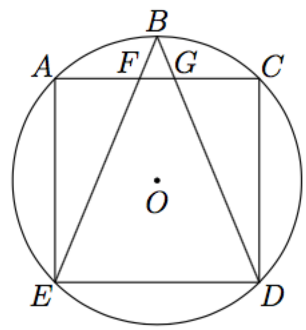
5. Maggie graphs the six possible lines of the form $y = mx + b$ where m is either 1 or -2, and b is either 0, 1 or 2. For example, one of the lines is $y = x + 2$. The lines are all graphed on the same axes. There are exactly n distinct points, each of which lies on two or more of these lines. What is the value of n?

6. How many perfect squares greater than 1 are divisors of 60^5?

7. Twenty-seven unit cubes are each coloured completely black or completely red. The unit cubes are assembled into a larger cube. If $\frac{1}{3}$ of the surface area of the larger cube is red, what is the smallest number of unit cubes that could have been coloured red?

8. Gina's running app tracked her average rate in minutes per kilometre. After starting the app, Gina stood still for 15 seconds and then ran at a constant rate of 7 minutes per kilometre for the rest of the run. How many kilometres did Gina run between when her app showed her average rate as 7 minutes 30 seconds per kilometre and when it showed 7 minutes 5 seconds per kilometre?

9. Square $ACDE$ is inscribed in a circle centred at O with radius $\sqrt{2}$. Point B is on the circumference of the circle so that BE and BD have the same length. Line segment AC intersects BE and BD at F and G, respectively. Determine the ordered pair (a, b) of integers so that the area of $\triangle BFG$ equals $a + b\sqrt{2}$.



10. Find all quadruples (a, b, c, d) of positive integers which satisfy

$$ab + 2a - b = 58$$
$$bc + 4b + 2c = 300$$
$$cd - 6c + 4d = 101$$

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PROBLEMS:

1. What is the integer that is greater than $\sqrt{11}$ but less than $\sqrt{19}$?
2. What is the value of $\dfrac{3^5 - 3^4}{3^3}$?
3. The measures of the interior angles of a quadrilateral are in the ratio $1 : 2 : 3 : 4$. Determine the measure in degrees of the smallest interior angle.
4. Determine the integer equal to

$$2020 + 1919 + 1818 + 1717 + 1616 + 1515 + 1414 + 1313 + 1212 + 1111 + 1010 .$$

5. What is the smallest eight-digit positive integer that has exactly four digits which are 4?
6. On Fridays, the price of a ticket to a museum is \$9. On one particular Saturday, there were 200 visitors to the museum, which was twice as many visitors as there were the day before. The total money collected from ticket sales on that particular Saturday was $\frac{4}{3}$ as much as the day before. The price of tickets on Saturdays is \$$k$. Determine the value of k.
7. What is the smallest four-digit positive integer that is divisible by both 5 and 9 and has only even digits?
8. The figure below was constructed by taking a semicircle with diameter 64 and replacing the diameter with four semicircles each having equal diameter. What is the perimeter of the figure?

9. How many times does the digit 0 appear in the integer equal to 20^{10}?
10. If $3y - 2x = 4$, determine the value of $\dfrac{16^{x+1}}{8^{2y-1}}$.

11. Let p_i be the i^{th} prime number; for example, $p_1 = 2, p_2 = 3$, and $p_3 = 5$. For each prime number, construct the point $Q_i(p_i, 0)$. Suppose A has coordinates $(0, 2)$. Determine the sum of the areas of the triangles $\triangle AQ_1Q_2, \triangle AQ_2Q_3, \triangle AQ_3Q_4, \triangle AQ_4Q_5, \triangle AQ_5Q_6$, and $\triangle AQ_6Q_7$.

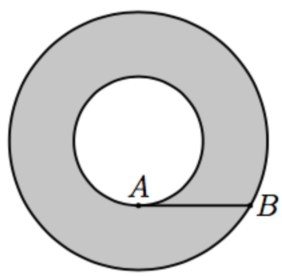
12. Éveriste listed all of the positive integers from 1 to 90. He then crossed out all of the multiples of 3 from the list. Of the remaining numbers, he then crossed out all of the multiples of 5. How many numbers were *not* crossed out?

13. The parabola with equation $y = -\frac{1}{4}x^2 + 5x - 21$ has its vertex at point A and crosses the x-axis at $B(b, 0)$ and $F(f, 0)$ where $b < f$. A second parabola with its vertex at B passes through A and crosses the y-axis at D. What are the coordinates of D?

14. Jeff caught 21 fish, each having a mass of at least 0.2 kg. He noticed that the average mass of the first three fish that he caught was the same as the average mass of all 21 fish. The total mass of the first three fish was 1.5 kg. What is the largest possible mass of any one fish that Jeff could have caught?

15. Suppose that a, b, c, and d are positive integers which are not necessarily distinct. If $a^2 + b^2 + c^2 + d^2 = 70$, what is the largest possible value of $a + b + c + d$?

16. In the diagram below, the two circles have the same centre. Point A is on the inner circle and point B is on the outer circle. Line segment AB has length 5 and is tangent to the inner circle at A. What is the area of the shaded region?



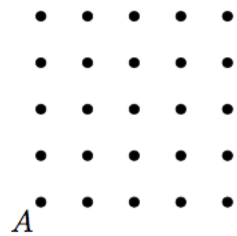
17. For each real number m, the function $f(x) = x^2 - 2mx + 8m + 4$ has a minimum value. What is the maximum of these minimum values?

18. A sequence $t_1, t_2, t_3, \ldots$ is defined by

$$t_n = \begin{cases} \dfrac{1}{7^n} & \text{when } n \text{ is odd} \\[2ex] \dfrac{2}{7^n} & \text{when } n \text{ is even} \end{cases}$$

for each positive integer n. Determine the sum of all of the terms in this sequence; that is, calculate $t_1 + t_2 + t_3 + \cdots$.

19. Pictured below is a rectangular array of dots with point the bottom left point labelled A. In how many ways can two points in the array be chosen so that they, together with point A, form a triangle with positive area?



20. Lyla and Isabelle run on a circular track both starting at point P. Lyla runs at a constant speed in the clockwise direction. Isabelle also runs in the clockwise direction at a constant speed 25% faster than Lyla. Lyla starts running first and Isabelle starts running when Lyla has completed one third of one lap. When Isabelle passes Lyla for the fifth time, how many times has Lyla returned to point P?

21. Suppose that $f(x) = 2\sin^2(\log x) + \cos(\log x^2) - 5$ for each $x > 0$. What is the value of $f(\pi)$?

22. How many triples (x, y, z) of positive integers satisfy both

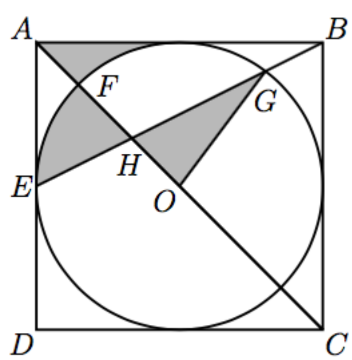
    (i) $x + y + z$ is a multiple of 3, and

    (ii) $1 \leq x \leq 10$, $1 \leq y \leq 10$, and $1 \leq z \leq 10$?

23. Rectangle $ABCD$ has diagonal BD with endpoints $B(4, 2)$ and $D(12, 8)$. Diagonal AC lies on the line with equation $x + 2y - 18 = 0$. Determine the area of $ABCD$.

24. Determine the sum of the real numbers x for which $\dfrac{2x}{x^2 + 5x + 3} + \dfrac{3x}{x^2 + x + 3} = 1$.

25. A circle with centre O is inscribed in square $ABCD$ having side length 60. Point E is the midpoint of AD. Line segments AC and BE intersect the top half of the circle at points F and G respectively, and they intersect each other at point H. What is the total area of the shaded regions?



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Relay Problem #0 (Seat a)

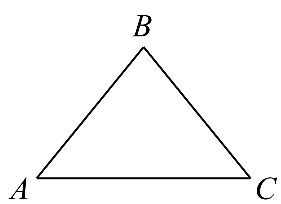
Evaluate $\dfrac{2+5\times 5}{3}$.

Relay Problem #0 (Seat b)

Let t be TNYWR.
What is the area of a triangle with base $2t$ and height $2t-6$?

Relay Problem #0 (Seat c)

Let t be TNYWR.
In the diagram, $\triangle ABC$ is isosceles with $AB = BC$. If $\angle ABC = t^\circ$, what is the measure of $\angle BAC$, in degrees?



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Relay Problem #1 (Seat a)

An equilateral triangle has sides of length $x + 5$, $y + 11$, and 14. What is the value of $x + y$?

Relay Problem #1 (Seat b)

Let t be TNYWR.
Gray has t dollars consisting of \$1 and \$2 coins. If she has the same number of \$1 and \$2 coins, how many \$1 coins does she have?

Relay Problem #1 (Seat c)

Let t be TNYWR.
Elise has t boxes, each containing x apples. She gives 10% of her apples to her brother. She then gives 6 apples to her sister. After this, she has 48 apples left. What is the value of x?

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Relay Problem #2 (Seat a)

The numbers $x+5$, 14, x, and 5 have an average of 9. What is the value of x?

Relay Problem #2 (Seat b)

Let t be TNYWR.
Each of the three lines having equations $x+ty+8=0$, $5x-ty+4=0$, and $3x-ky+1=0$ passes through the same point. What is the value of k?

Relay Problem #2 (Seat c)

Let t be TNYWR.
Quadrilateral $ABCD$ has vertices $A(0,3)$, $B(0,k)$, $C(t,10)$, and $D(t,0)$, where $k>3$ and $t>0$. The area of quadrilateral $ABCD$ is 50 square units. What is the value of k?

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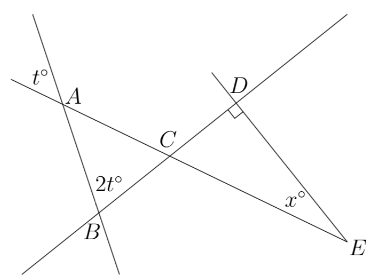
Relay Problem #3 (Seat a)

Let M be the number of multiples of 5 between 1 to 2020 inclusive and N be the number of multiples of 20 between 1 and 2020 inclusive. What is the value of $10M \div N$.

Relay Problem #3 (Seat b)

Let t be TNYWR.
Four line segments intersect in points A, B, C, D, and E, as shown. The measure of $\angle CED$ is x°. What is the value of x?



Relay Problem #3 (Seat c)

Let t be TNYWR.
Armen paid \$190 to buy movie tickets for a group of t people, consisting of some adults and some children. Movie tickets cost \$5 for children and \$9 for adults. How many children's tickets did he buy?