

The CENTRE for EDUCATION in MATHEMATICS and COMPUTING cemc.uwaterloo.ca

2020 Canadian Team Mathematics Contest

Individual Problems

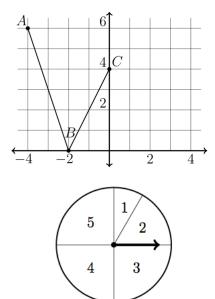
IMPORTANT NOTES:

- Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) previously stored information such as formulas, programs, notes, etc., (iv) a computer algebra system, (v) dynamic geometry software.
- Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 \sqrt{2}$ are simplified exact numbers.

PROBLEMS:

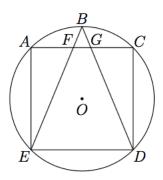
- 1. What is the value of $\frac{24+12}{4^2-4}$?
- 2. If 3k = 10, what is the value of $\frac{6}{5}k 2$?
- 3. Segment AB is reflected in the y-axis to obtain ED. Segment BC is reflected in the y-axis to obtain DC. Determine the sum of the slopes of DC and ED.

4. A spinner was created by drawing five radii from the centre of a circle. The first four radii divide the circle into four equal wedges. The fifth radius divides one of the wedges into two parts, one having twice the area of the other. The five wedges are labelled as pictured with the wedge labeled by 2 having twice the area of the wedge labeled by 1. Determine the probability of spinning an odd number.



- 5. Maggie graphs the six possible lines of the form y = mx + b where m is either 1 or -2, and b is either 0, 1 or 2. For example, one of the lines is y = x + 2. The lines are all graphed on the same axes. There are exactly n distinct points, each of which lies on two or more of these lines. What is the value of n?
- 6. How many perfect squares greater than 1 are divisors of 60^5 ?
- 7. Twenty-seven unit cubes are each coloured completely black or completely red. The unit cubes are assembled into a larger cube. If $\frac{1}{3}$ of the surface area of the larger cube is red, what is the smallest number of unit cubes that could have been coloured red?

- 8. Gina's running app tracked her average rate in minutes per kilometre. After starting the app, Gina stood still for 15 seconds and then ran at a constant rate of 7 minutes per kilometre for the rest of the run. How many kilometres did Gina run between when her app showed her average rate as 7 minutes 30 seconds per kilometre and when it showed 7 minutes 5 seconds per kilometre?
- 9. Square ACDE is inscribed in a circle centred at O with radius $\sqrt{2}$. Point B is on the circumference of the circle so that BE and BD have the same length. Line segment AC intersects BE and BD at F and G, respectively. Determine the ordered pair (a, b) of integers so that the area of $\triangle BFG$ equals $a + b\sqrt{2}$.



10. Find all quadruples (a, b, c, d) of positive integers which satisfy

$$ab + 2a - b = 58$$
$$bc + 4b + 2c = 300$$
$$cd - 6c + 4d = 101$$