The CENTRE for EDUCATION
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2020 Canadian Intermediate Mathematics Contest

Wednesday, November 18, 2020
(in North America and South America)

Thursday, November 19, 2020
(outside of North America and South America)

Solutions

Part A

1. We write each of the three given fractions as a decimal:

$$\frac{1}{4} = 0.25 \qquad \frac{4}{10} = 0.4 \qquad \frac{41}{100} = 0.41$$

The five numbers are thus 0.25, 0.4, 0.41, 0.04, 0.404.
Re-writing this list in increasing order, we obtain 0.04, 0.25, 0.4, 0.404, 0.41.
Therefore, the middle number is 0.4 or $\frac{4}{10}$.

ANSWER: $\frac{4}{10}$

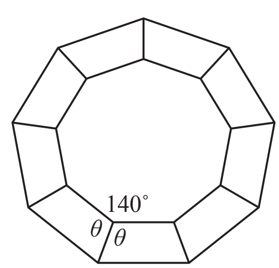
2. Each 8 by 10 rectangle has area $8 \times 10 = 80$.
The 4 by 4 square has area $4 \times 4 = 16$.
Each of the two shaded pieces is the part of an 8 by 10 rectangle outside a 4 by 4 square, and so has area $80 - 16 = 64$.
Therefore, the total area of the shaded region is $2 \times 64 = 128$.
(The area of the shaded region could also be determined by dividing it into rectangular pieces.)

ANSWER: 128

3. After 10 days, Juan has removed $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55$ candies.
On day 11, Juan removes 11 candies so has now removed $55 + 11 = 66$ candies.
This means that, after day 10, Juan has removed fewer than 64 candies, and, after day 11, Juan has removed more than 64 candies.
Therefore, the smallest possible value of n is 11.

ANSWER: 11

4. *Solution 1*
The inner polygon formed by the nine trapezoids is a regular polygon with 9 sides. This is because (i) all of the sides of this polygon are equal in length (equal to the shorter parallel side of one of the trapezoids), and (ii) the measure of each of the interior angles of this polygon is equal (formed between two identical trapezoids).
The sum of the measures of the angles in this polygon with 9 sides is $180^\circ(9 - 2)$ or 1260°.
Thus, the measure of each of the interior angles is $1260^\circ \div 9 = 140^\circ$.
Let $\theta = \angle ABC$.
Since the nine trapezoids are identical, then the trapezoid adjacent to the left also has an angle of θ, as shown:



Since the sum of the measures of angles around a point is 360°, then $2\theta + 140^\circ = 360^\circ$ and so $2\theta = 220^\circ$ which gives $\angle ABC = \theta = 110^\circ$.

Solution 2

The outer polygon formed by the nine trapezoids is a regular polygon with 9 sides. This is because (i) all of the sides of this polygon are equal in length (equal to the longer parallel side of one of the trapezoids), and (ii) the measure of each of the interior angles of this polygon is equal (formed by two of the base angles of two identical trapezoids.)

The sum of the measures of the angles in this polygon with 9 sides is $180^\circ(9-2)$ or 1260°.

Thus, the measure of each of the interior angles is $1260^\circ \div 9 = 140^\circ$.

Each of these interior angles is made up from two equal base angles, which are thus equal to one-half of 140° or 70°.

Since the trapezoids have parallel sides, then $\angle ABC$ is supplementary to one of these base angles, and so $\angle ABC = 180^\circ - 70^\circ = 110^\circ$.

ANSWER: 110°

5. Throughout this solution, we use the fact that if a and b are positive, then $\frac{1}{a}$ is less than $\frac{1}{b}$ exactly when a is greater than b. (That is, $\frac{1}{a} < \frac{1}{b}$ exactly when $a > b$.)

Since $\frac{1}{x} + \frac{1}{y}$ is positive, then x and y cannot both be negative (otherwise $\frac{1}{x} + \frac{1}{y}$ would be negative).

Since $x \leq y$, this means that either both x and y are positive or x is negative and y is positive.

Suppose that x and y are both positive. This means that $\frac{1}{x}$ and $\frac{1}{y}$ are both positive.

Since $\frac{1}{x} + \frac{1}{y} = \frac{1}{4}$, then $\frac{1}{x}$ and $\frac{1}{y}$ are both less than $\frac{1}{4}$.

This means that $x > 4$ and $y > 4$.

Since $x \leq y$, then $\frac{1}{x} \geq \frac{1}{y}$ which means that $\frac{1}{4} = \frac{1}{x} + \frac{1}{y} \leq \frac{1}{x} + \frac{1}{x}$ or $\frac{1}{4} \leq \frac{2}{x}$ which gives $\frac{2}{8} \leq \frac{2}{x}$ and so $x \leq 8$. (In other words, since $\frac{1}{x}$ is larger, then it must be at least one-half of $\frac{1}{4}$ so $\frac{1}{x}$ is at least $\frac{1}{8}$ and so x is at most 8.)

Since $x > 4$ and $x \leq 8$, then x could equal 5, 6, 7, or 8.

If $x = 5$, then $\frac{1}{y} = \frac{1}{4} - \frac{1}{5} = \frac{5}{20} - \frac{4}{20} = \frac{1}{20}$ and so $y = 20$.

If $x = 6$, then $\frac{1}{y} = \frac{1}{4} - \frac{1}{6} = \frac{3}{12} - \frac{2}{12} = \frac{1}{12}$ and so $y = 12$.

If $x = 7$, then $\frac{1}{y} = \frac{1}{4} - \frac{1}{7} = \frac{7}{28} - \frac{4}{28} = \frac{3}{28}$ and so $y = \frac{28}{3}$ which is not an integer.

If $x = 8$, then $\frac{1}{y} = \frac{1}{4} - \frac{1}{8} = \frac{2}{8} - \frac{1}{8} = \frac{1}{8}$ and so $y = 8$.

Therefore, when x and y are both positive, the solutions are $(x, y) = (5, 20), (6, 12), (8, 8)$.

Suppose that x is negative and y is positive. This means that $\frac{1}{x}$ is negative and $\frac{1}{y}$ is positive.

Since $\frac{1}{x} + \frac{1}{y} = \frac{1}{4}$, then $\frac{1}{y}$ is greater than $\frac{1}{4}$.

This means that $y < 4$.

Since y is positive and $y < 4$, then y could equal 1, 2 or 3.

If $y = 1$, then $\frac{1}{x} = \frac{1}{4} - \frac{1}{1} = \frac{1}{4} - \frac{4}{4} = -\frac{3}{4}$ and so $x = -\frac{4}{3}$, which is not an integer.
If $y = 2$, then $\frac{1}{x} = \frac{1}{4} - \frac{1}{2} = \frac{1}{4} - \frac{2}{4} = -\frac{1}{4}$ and so $x = -4$.
If $y = 3$, then $\frac{1}{x} = \frac{1}{4} - \frac{1}{3} = \frac{3}{12} - \frac{4}{12} = -\frac{1}{12}$ and so $x = -12$.
Therefore, when x is negative and y is positive, the solutions are $(x, y) = (-4, 2), (-12, 3)$.

In summary, the solutions are $(x, y) = (-4, 2), (-12, 3), (5, 20), (6, 12), (8, 8)$.

ANSWER: $(x, y) = (-4, 2), (-12, 3), (5, 20), (6, 12), (8, 8)$

6. When each of 90 players plays against all of the other 89 players exactly once, the total number of games played is $\frac{90 \times 89}{2} = 4005$.
This is because each of the 90 players plays 89 games, giving 90×89, but each of the games is counted twice in this total, since each game is played by two players, giving $\frac{90 \times 89}{2}$.
Exactly 1 point is awarded in total for each game. This is because the two players either win and lose $(1 + 0$ points) or both tie $(0.5 + 0.5$ points).
Therefore, over 4005 games, a total of 4005 points are awarded to the players in the league.
Suppose that, after all games have been played, n players have at least 54 points.
Then $54n \leq 4005$ since these n players cannot have more points than the total number of points awarded to the entire league.
Since $\frac{4005}{54} \approx 74.17$ and n is an integer, this means that $n \leq 74$. (Note that $74 \times 54 = 3996$ and $75 \times 54 = 4050$.)

Suppose that 74 players have at least 54 points.
This accounts for at least $74 \times 54 = 3996$ of the total of 4005 points.
This leaves at most $4005 - 3996 = 9$ points to be distributed among the remaining $90 - 74 = 16$ players.
These 16 players play $\frac{16 \times 15}{2} = 120$ games amongst themselves (each of 16 players plays the other 15 players) and these games generate 120 points to be distributed among the 16 players.
This means that these 16 players have at least 120 points between them, and possibly more if they obtain points when playing the other 54 players.
Therefore, these 16 players have at most 9 points and at least 120 points in total, which cannot happen.
Thus, it cannot be the case that 74 players have at least 54 points.

Suppose that 73 players have at least 54 points.
This accounts for at least $73 \times 54 = 3942$ of the total of 4005 points.
This leaves at most $4005 - 3942 = 63$ points to be distributed among the remaining $90 - 73 = 17$ players.
These 17 players play $\frac{17 \times 16}{2} = 136$ games amongst themselves.
This means that these 17 players have at least 136 points between them.
Therefore, these 17 players have at most 63 points and at least 136 points in total, which cannot happen.
Thus, it cannot be the case that 73 players have at least 54 points.

Suppose that 72 players have at least 54 points.
This accounts for at least $72 \times 54 = 3888$ of the total of 4005 points.

This leaves at most $4005 - 3888 = 117$ points to be distributed among the remaining $90 - 72 = 18$ players.
These 18 players play $\frac{18 \times 17}{2} = 153$ games amongst themselves.
This means that these 18 players have at least 153 points between them.
Therefore, these 18 players have at most 117 points and at least 153 points in total, which cannot happen.
Thus, it cannot be the case that 72 players have at least 54 points.

Suppose that 71 players have at least 54 points.
This accounts for at least $71 \times 54 = 3834$ of the total of 4005 points.
This leaves at most $4005 - 3834 = 171$ points to be distributed among the remaining $90 - 71 = 19$ players.
These 19 players play $\frac{19 \times 18}{2} = 171$ games amongst themselves.
This means that these 19 players have at least 171 points between them.
These point totals match. Therefore, it appears that 71 players could have at least 54 points.

It is in fact possible for this to happen.
Suppose that every game between two of the 71 players ends in a tie, every game between one of the 71 players and one of the 19 players ends in a win for the first player, and every game between two of the 19 players ends in a tie.
In this case, each of the 71 players has 70 ties and 19 wins, for a point total of $(70 \times 0.5) + (19 \times 1) = 54$, as required.

ANSWER: 71

Part B

1. (a) In the arithemetic sequence $20, 13, 6, -1$, each term is obtained by adding a constant. This constant is $13 - 20 = -7$.
Therefore, the next two terms are $-1 + (-7) = -8$ and $-8 + (-7) = -15$.

(b) The arithmetic sequence $2, a, b, c, 14$ has 5 terms.
This means that the constant difference (that is, the constant that is added to one term to obtain the next) is added 4 times to the first term (which equals 2) to obtain the last term (which equals 14).
Thus, the constant difference must be $\dfrac{14-2}{4} = \dfrac{12}{4} = 3$.
This means that the sequence starts at 2 and counts by 3s: 2, 5, 8, 11, 14.
Therefore, $a = 5$, $b = 8$, and $c = 11$.

(c) There are six possible ways in which the three terms can be arranged:

$$7, 15, t \qquad 15, 7, t \qquad 7, t, 15 \qquad 15, t, 7 \qquad t, 7, 15 \qquad t, 15, 7$$

We determine the corresponding value of t in each case:

- $7, 15, t$: Here, the constant difference is $15 - 7 = 8$. Thus, $t = 15 + 8 = 23$.
- $15, 7, t$: Here, the constant difference is $7 - 15 = -8$. Thus, $t = 7 + (-8) = -1$.
- $t, 7, 15$: Here, the constant difference is $15 - 7 = 8$. Thus, $t + 8 = 7$ and so $t = -1$.
- $t, 15, 7$: Here, the constant difference is $7 - 15 = -8$. Thus, $t + (-8) = 15$ and so $t = 23$.
- $7, t, 15$: Here, the constant difference is added 2 times to 7 to get 15. Thus, the constant difference is $\dfrac{15-7}{2} = \dfrac{8}{2} = 4$. Thus, $t = 7 + 4 = 11$.
- $15, t, 7$: Here, the constant difference is $\dfrac{7-15}{2} = \dfrac{-8}{2} = -4$. Thus, $t = 15+(-4) = 11$.

Therefore, the possible values of t are 23, -1, and 11.

(d) Suppose that the sequence r, s, w, x, y, z has constant difference d.
This means that the difference between any two consecutive terms is d.
Since the sequence has 6 terms, then the difference is added 5 times to r to get z. In other words, $z - r$ is equal to $5d$.
Also, the difference between a later term in the sequence and an earlier term in the sequence must equal one of d, $2d$, $3d$, $4d$, or $5d$, depending on the number of terms between the two specific terms.
Since 4 and 20 are two terms in the sequence, with 20 appearing later than 4, and $20 - 4 = 16$, then either $d = 16$ or $2d = 16$ (which gives $d = 8$) or $3d = 16$ (which gives $d = \dfrac{16}{3}$) or $4d = 16$ (which gives $d = 4$) or $5d = 16$ (which gives $d = \dfrac{16}{5}$).
The corresponding values of $5d$ are 80, 40, $\dfrac{80}{3}$, 20, and 16.
Since $z - r$ is equal to $5d$, then the largest possible value of $z - r$ is 80 and the smallest possible value is 16.

(The sequence $4, 20, 36, 52, 68, 84$ shows that a difference of 80 is possible.
The sequence $4, \frac{36}{5}, \frac{52}{5}, \frac{68}{5}, \frac{84}{5}, 20$ shows that a difference of 16 is possible.)

2. (a) (i) The volume of Tank B is $5 \text{ cm} \times 9 \text{ cm} \times 8 \text{ cm} = 360 \text{ cm}^3$.
This means that Tank B is $\frac{1}{3}$ full when it contains $\frac{1}{3} \times 360 \text{ cm}^3 = 120 \text{ cm}^3$ of water.
Since Tank B fills at $4 \text{ cm}^3/\text{s}$, then Tank B is $\frac{1}{3}$ full after $\dfrac{120 \text{ cm}^3}{4 \text{ cm}^3/\text{s}} = 30$ s.

(ii) Since the volume of Tank B is 360 cm^3 and it fills at $4 \text{ cm}^3/\text{s}$, then it is full after $\dfrac{360 \text{ cm}^3}{4 \text{ cm}^3/\text{s}} = 90$ s.
(We can also note that since it is $\frac{1}{3}$ full after 30 s and it fills at a constant rate, then it is completely full after $3 \times 30 \text{ s} = 90$ s.)
Since Tank A drains at $4 \text{ cm}^3/\text{s}$, then in 90 s, 360 cm^3 of water has drained out.
Since the volume of Tank A is $10 \text{ cm} \times 8 \text{ cm} \times 6 \text{ cm} = 480 \text{ cm}^3$, then after 90 s, the volume of water in Tank A is $480 \text{ cm}^3 - 360 \text{ cm}^3 = 120 \text{ cm}^3$.
Tank A is sitting on one of its $10 \text{ cm} \times 8 \text{ cm}$ faces, which has an area of 80 cm^2.
Since the water itself forms a rectangular prism, the depth of water in Tank A at this instant is $\dfrac{120 \text{ cm}^3}{80 \text{ cm}^2} = 1.5$ cm.

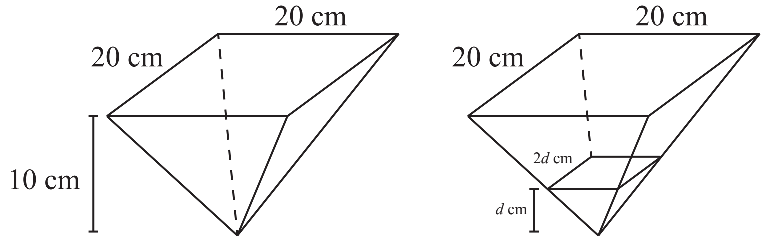
(iii) Since Tank A starts full and Tank B starts empty and the rate at which water leaves Tank A equals the rate at which water enters Tank B, then the combined volume of water in the two tanks is constant and equal to the initial volume of Tank A, or 480 cm^3.
Let the depth of water in Tank A and Tank B when their depths are equal be d cm.
Since the area of the bottom face of Tank A is 80 cm^2, then the volume of water in Tank A when the depth is d cm is $80d \text{ cm}^3$.
Since the area of the bottom face of Tank B is $5 \text{ cm} \times 9 \text{ cm} = 45 \text{ cm}^2$, then the volume of water in Tank B when the depth is d cm is $45d \text{ cm}^3$.
Since the combined volume is 480 cm^3, then $80d + 45d = 480$ which gives $125d = 480$ or $25d = 96$ and so $d = \frac{96}{25} = 3.84$.
Thus, the depth is 3.84 cm.

(b) Suppose that the volumes of water in Tank C and Tank D are equal t seconds after Tank D begins to fill.
The volume of Tank C is $31 \text{ cm} \times 4 \text{ cm} \times 4 \text{ cm} = 496 \text{ cm}^3$.
Since Tank C starts full, it drains at $2 \text{ cm}^3/\text{s}$, and will have drained for $t - 2$ seconds, then the volume of water in Tank C after t seconds is $(496 - 2(t - 2)) \text{ cm}^3$.
Since Tank D starts empty and fills at $1 \text{ cm}^3/\text{s}$, then the volume of water in Tank D after t seconds is $t \text{ cm}^3$.
Since these volumes are equal, then

$$\begin{aligned} 496 - 2(t-2) &= t \\ 496 - 2t + 4 &= t \\ 500 &= 3t \\ t &= \tfrac{500}{3} \end{aligned}$$

Therefore, the volumes are equal after $\frac{500}{3}$ seconds. At this time, the volume of water in Tank D is $\frac{500}{3} \text{ cm}^3$.

Suppose that the depth of water in Tank D at this instant is d cm. Note that the water forms a square-based pyramid with its top pointing downwards. The surface of the water in Tank D will be a square with side length $2d$ cm.



This is because, as Tank D fills, the ratio of the side length of the base to the height will remain constant. This ratio for the whole tank is 20 cm : 10 cm which is equivalent to 2 : 1. We could say that the square-based pyramid formed by the water at any instant is *similar* to the Tank itself.

The volume of a square-based pyramid whose base has an edge length of $2d$ cm and whose height is d cm is $\frac{1}{3}(2d)^2 d$ cm^3, which equals $\frac{4}{3}d^3$ cm^3.

Therefore, $\frac{4}{3}d^3 = \frac{500}{3}$ which gives $d^3 = 125$ and so $d = 5$.

Therefore, at the instant when the volume of water in Tank C equals the volume of water in Tank D, the depth of water in Tank D is 5 cm.

3. (a) P can be a multiple of 216.
For example, if $a = 6$, $b = 6$, $c = 6$, and $d = 2$, then $P = abcd = 432$ which equals 2×216.

(b) P cannot be a multiple of 2000.
To see this, we note first that $2000 = 2 \times 10 \times 10 \times 10 = 2^4 \times 5^3$.
For $P = abcd$ to be a multiple 2000, P would need to include at least 4 factors of 2 and at least 3 factors of 5.
Thus the result of the product $abcd$ would need to include at least 3 factors of 5.
Since the only values for a, b, c, d are $1, 2, 3, 4, 5, 6, 7, 8, 9$, the only way to introduce factors of 5 are for one or more of the integers a, b, c, d to equal 5. There are no other multiples of 5 in this list.
To be a multiple of 2000, P must have 3 factors of 5, so at least 3 of a, b, c, d equal 5.
Suppose that $a = b = c = 5$. Then $P = 5 \times 5 \times 5 \times d = 125d$.
In this case, P is at most $125 \times 9 = 1125$ which is too small to be a multiple of 2000.
Alternatively, we could note that d cannot include the required 4 factors of 2 since there are no multiples of $2^4 = 16$ in the list.
Therefore, P cannot be a multiple of 16 and thus cannot be a multiple of 2000.

(c) We note first that $2^7 = 128$ and $2^{10} = 1024$.
For P to be divisble by $2^7 = 128$ but not by $2^{10} = 1024$, the integer P needs to have at least 7 factors of 2 but fewer than 10 factors of 2. In other words, P must be divisible by 2^7, could be divisible by 2^8 or by 2^9, but cannot be divisible by any larger power of 2.
Assume that a has A factors of 2, b has B factors of 2, c has C factors of 2, and d has D factors of 2. Therefore, $7 \leq A + B + C + D \leq 9$.
We note that we need to count possible values of P, not quadruples (a, b, c, d).
As a result, we may assume that $A \geq B \geq C \geq D$. In other words, a has at least as many factors of 2 as b, b has at least as many factors of 2 as c, and so on.
Among the possible values of a, b, c, d, there is one that includes exactly 3 factors of 2 (namely, 8), and one that includes exactly 2 factors of 2 (namely, 4), and two that include

exactly 1 factor of 2 (namely, 2 and 6).
Therefore, each of A, B, C, D is at least 0 and at most 3.
Suppose $A+B+C+D=9$. Since $3 \geq A \geq B \geq C \geq D \geq 0$, then we have the following possibilities:

$$3+3+3+0=9 \qquad 3+3+2+1=9 \qquad 3+2+2+2=9$$

In each case, we determine the possible values of P:

- $3+3+3+0=9$: Here, $a=b=c=8$ and d is one of 1, 3, 5, 7, or 9, which gives 5 possible values of P: 2^9, 3×2^9, 5×2^9, 7×2^9, 9×2^9.
- $3+3+2+1=9$: Here, $a=b=8$ and $c=4$ and $d=2$ or $d=6$. There are no additional values of P here, as we get 2^9 and 3×2^9 as possible values for P.
- $3+2+2+2=9$: Here, $a=8$ and $b=c=d=4$. There are no additional values of P here, as the only possible value of P here is 2^9.

Suppose $A+B+C+D=8$. Since $3 \geq A \geq B \geq C \geq D \geq 0$, then we have the following possibilities:

$$3+3+2+0=8 \qquad 3+3+1+1=8 \qquad 3+2+2+1=8 \qquad 2+2+2+2=8$$

In each case, we determine the possible values of P:

- $3+3+2+0=8$: Here, $a=b=8$ and $c=4$ and d is one of 1, 3, 5, 7, or 9, which gives 5 additional values of P: 2^8, 3×2^8, 5×2^8, 7×2^8, 9×2^8.
- $3+3+1+1=8$: Here, $a=b=8$ and c and d each equal 2 or 6. This means that P can equal 2^8 or 3×2^8 or 9×2^8, which give no additional values of P.
- $3+2+2+1=8$: Here, $a=8$ and $b=c=4$ and $d=2$ or $d=6$. There are no additional values of P here.
- $2+2+2+2=8$: Here, $a=b=c=d=4$. There are no additional values of P here.

Suppose $A+B+C+D=7$. Since $3 \geq A \geq B \geq C \geq D \geq 0$, then we have the following possibilities:

$$3+3+1+0=7 \qquad 3+2+2+0=7 \qquad 3+2+1+1=7 \qquad 2+2+2+1=7$$

In each case, we determine the possible values of P:

- $3+3+1+0=7$: Here, $a=b=8$ and $c=2$ or $c=6$ and d is one of 1, 3, 5, 7, or 9. From these, we get eight additional values of P: 2^7, 3×2^7, 5×2^7, 7×2^7, 9×2^7, 15×2^7, 21×2^7, 27×2^7.
- $3+2+2+0=7$: Here, $a=8$ and $b=c=4$ and $d=1,3,5,7,9$. There are no additional values of P here.
- $3+2+1+1=7$: Here, $a=8$ and $b=4$ and each of c and d is 2 or 6. There are no additional values of P here.
- $2+2+2+1=7$: Here, $a=b=c=4$ and d is 2 or 6. There are no additional values of P here.

In total, there are $5+5+8=18$ possible values of P.

(d) Since P is 98 less than a multiple of 100, then $P = 2$ or the last two digits of P are 02.
A positive integer is divisible by 5 exactly when its units digit is either 0 or 5. Therefore, P is not divisible by 5.
A positive integer is divisible by 4 exactly when the integer formed by its last two digits is divisible by 4. Since 2 is not divisible by 4, P is not divisible by 4.
Since $P = abcd$ and each of a, b, c, d is equal to one of 1, 2, 3, 4, 5, 6, 7, 8, 9, this means that

- none of a, b, c, d equals 5 because P is not a multiple of 5, and
- none of a, b, c, d equals 4 or 8 because P is not a multiple of 4, and
- exactly one of a, b, c, d equals 2 or 6 because P is even but not a multiple of 4.

In other words, exactly one of a, b, c, d equals 2 or 6 and the remaining variables are restricted to being equal to 1, 3, 7, or 9.
For the moment, we suppose that a equals 2 or 6 and that $b \leq c \leq d$. We will eventually lift these restrictions in order to be able to arrange the possible values into ordered quadruples.

Suppose that $a = 2$.
Since P equals 2 or ends in 02, this means that $P = 100k+2$ for some non-negative integer k.
Since $P = abcd$ and $a = 2$, then $2 \times bcd = 100k + 2$ or $bcd = 50k + 1$.
Thus, bcd is 1 more than a multiple of 50, which means that $bcd = 1$ or the final two digits of bcd are 01 or 51.

Suppose that $a = 6$. Here, $P = 6bcd$. Note also that $P > 2$ in this case which means that $P \geq 102$.
Since P has a units digit of 2 and $P = 6 \times bcd$, the units digit of bcd must be 2 or 7.
We write $bcd = 10q + 2$ or $bcd = 10q + 7$ for some non-negative integer q.
Thus, $6 \times bcd = 60q + 12$ or $6 \times bcd = 60q + 42$ and so $60q = P - 12$ or $60q = P - 42$.
Since the final two digits of P are 02, the final two digits of $60q$ are either 90 or 60.
Since 60 is divisible by 4, then $60q$ is divisible by 4. Since an integer ending in 90 is not divisible by 4, this case is not possible and so we cannot have $bcd = 10q + 2$.
If the final two digits of $60q = 10 \times (6q)$ are 60, then $6q$ has a units digit of 6, and so q must have a units digit of 1 or 6. (We can see this by testing all of the possible units digits of q.)
Since $bcd = 10q + 7$, then the final two digits of bcd are 17 or 67.

We now need to determine what, if any, products bcd have final two digits 01, 51, 17, 67, where $b \leq c \leq d$, each of b, c, d equals one of 1, 3, 7, 9, and $a = 2$ (digits 01 or 51) or $a = 6$ (digits 17 or 67).
Suppose that $b = c = 1$ and so $bcd = d$. The only possibility that gives the correct digits is $d = 1$, which corresponds to $a = 2$.
Suppose that $b = 1$ and $c > 1$. Then $bcd = cd$. Here, we have the following possibilities for cd:

$$3 \times 3 = 9 \quad 3 \times 7 = 21 \quad 3 \times 9 = 27 \quad 7 \times 7 = 49 \quad 7 \times 9 = 63 \quad 9 \times 9 = 81$$

We do not get the correct digits here.
Suppose that $b = 3$ and so $bcd = 3 \times cd$. Since $3 \leq c \leq d$, we can use the possibilities from the previous case. Here, we can multiply the previous possibilities by 3 to see that bcd can be equal to any of 27, 63, 81, 147, 189, 243, none of which has the correct digits.
Suppose that $b = 7$ and so $7 \leq c \leq d$. The possibilities here are:

$$7 \times 7 \times 7 = 343 \quad 7 \times 7 \times 9 = 441 \quad 7 \times 9 \times 9 = 567$$

The last case gives the correct digits for P when $a = 6$.
Finally, if $b = 9$, then we must have $b = c = d = 9$ and so $bcd = 729$, which does not end with the correct digits.

Therefore, we could have $a = 2$ with $b = c = d = 1$ or $a = 6$ with $b = 7$ and $c = d = 9$.

Finally, we arrange $2, 1, 1, 1$ and $6, 7, 9, 9$ into ordered quadruples.
In the first case, there are 4 spots into which one can place the 2, and so there are 4 ordered quadruples in this case.
In the second case, there are 4 spots into which to first put the 6 and then 3 remaining spots into which to put the 7, leaving the 9s to be put without choice into the remaining spots. This gives $4 \times 3 = 12$ ordered quadruples.
In total, there are $4 + 12 = 16$ ordered quadruples (a, b, c, d) for which the product P is 98 less than a multiple of 100.