The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

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2019 Canadian Team Mathematics Contest

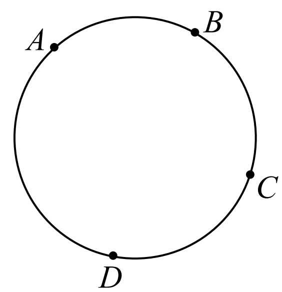
Individual Problems

IMPORTANT NOTES:

- Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) previously stored information such as formulas, programs, notes, etc., (iv) a computer algebra system, (v) dynamic geometry software.
- Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.

PROBLEMS:

1. In the diagram, points A, B, C, and D are on a circle. Philippe uses a ruler to connect each pair of these points with a line segment. How many line segments does he draw?



2. What is the smallest positive integer n for which $\sqrt{2019 - n}$ is an integer?
3. At 7:00 a.m. yesterday, Sherry correctly determined what time it had been 100 hours before. What was her answer? (Be sure to include "a.m." or "p.m." in your answer.)
4. A standard die with six faces is tossed onto a table. Itai counts the total number of dots on the five faces that are not lying on the table. What is the probability that this total is at least 19?
5. Suppose that a, b, c, and d are positive integers with $0 < a < b < c < d < 10$. What is the maximum possible value of $\dfrac{a-b}{c-d}$?
6. When the line with equation $y = -2x + 7$ is reflected across the line with equation $x = 3$, the equation of the resulting line is $y = ax + b$. What is the value of $2a + b$?
7. Suppose that $(2^3)^x = 4096$ and that $y = x^3$. What is the ones (units) digit of the integer equal to 3^y?
8. Yasmine makes her own chocolate beverage by mixing volumes of milk and syrup in the ratio $5 : 2$. Milk comes in 2 L bottles and syrup comes in 1.4 L bottles. Yasmine has a limitless supply of full bottles of milk and of syrup. Determine the smallest volume of chocolate beverage that Yasmine can make that uses only whole bottles of both milk and syrup.

9. Suppose that a is an integer. A sequence $x_1, x_2, x_3, x_4, \ldots$ is constructed with

- $x_1 = a$,
- $x_{2k} = 2x_{2k-1}$ for every integer $k \geq 1$, and
- $x_{2k+1} = x_{2k} - 1$ for every integer $k \geq 1$.

For example, if $a = 2$, then

$$x_1 = 2 \qquad x_2 = 2x_1 = 4 \qquad x_3 = x_2 - 1 = 3 \qquad x_4 = 2x_3 = 6 \qquad x_5 = x_4 - 1 = 5$$

and so on. The integer $N = 578$ can appear in this sequence after the 10th term (for example, $x_{12} = 578$ when $a = 10$), but the integer 579 does not appear in the sequence after the 10th term for any value of a. What is the smallest integer $N > 1395$ that could appear in the sequence after the 10th term for some value of a?

10. In the diagram, $ABCDEFGH$ is a rectangular prism. (Vertex H is hidden in this view.) If $\angle ABF = 40°$ and $\angle ADF = 20°$, what is the measure of $\angle BFD$, to the nearest tenth of a degree?