# 2017 Canadian Team Mathematics Contest Individual Problems 

## IMPORTANT NOTES:

- Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.
- Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi+1$ and $1-\sqrt{2}$ are simplified exact numbers.


## PROBLEMS:

1. What is the value of $x$ so that $\frac{8}{x}+6=8$ ?
2. In the diagram, $A B$ is parallel to $C D$. Points $E$ and $F$ are on $A B$ and $C D$, respectively, so that $\angle F A B=30^{\circ}$ and $\angle A F E=\angle E F B=\angle B F D=x^{\circ}$. What is the value of $x$ ?

3. The average height of Ivan, Jackie and Ken is $4 \%$ larger than the average height of Ivan and Jackie. If Ivan and Jackie are each 175 cm tall, how tall is Ken?
4. A positive integer $n$ between 10 and 99 , inclusive, is chosen at random. If every such integer is equally likely to be chosen, what is the probability that the sum of the digits of $n$ is a multiple of 7 ?
5. A car and a minivan drive from Alphaville to Betatown. The car travels at a constant speed of $40 \mathrm{~km} / \mathrm{h}$ and the minivan travels at a constant speed of $50 \mathrm{~km} / \mathrm{h}$. The minivan passes the car 10 minutes before the car arrives at Betatown. How many minutes pass between the time at which the minivan arrives in Betatown and the time at which the car arrives in Betatown?
6. Ruxandra wants to visit Singapore, Mongolia, Bhutan, Indonesia, and Japan. In how many ways can she order her trip to visit each country exactly once, with the conditions that she cannot visit Mongolia first and cannot visit Bhutan last?
7. Liesl has a bucket. Henri drills a hole in the bottom of the bucket. Before the hole was drilled, Tap A could fill the bucket in 16 minutes, tap B could fill the bucket in 12 minutes, and tap C could fill the bucket in 8 minutes. A full bucket will completely drain out through the hole in 6 minutes. Liesl starts with the empty bucket with the hole in the bottom and turns on all three taps at the same time. How many minutes will it take until the instant when the bucket is completely full?
8. In the diagram, the two regular octagons have side lengths of 1 and 2 . The smaller octagon is completely contained within the larger octagon. What is the area of the region inside the larger octagon and outside the smaller octagon?

9. The parabolas with equations $y=x^{2}-2 x-3$ and $y=-x^{2}+4 x+c$ intersect at points $A$ and $B$. Determine the value of $c$ so that the sum of the $x$-coordinate and $y$-coordinate of the midpoint of $A B$ is 2017 .
10. Determine the number of pairs of integers, $(a, b)$, with $1 \leq a \leq 100$ so that the line with equation $b=a x-4 y$ passes through point $(r, 0)$, where $r$ is a real number with $0 \leq r \leq 3$, and passes through point $(s, 4)$, where $s$ is a real number with $2 \leq s \leq 4$.
