



The CENTRE for EDUCATION in MATHEMATICS and COMPUTING



2017 Beaver Computing Challenge (Grade 9 & 10)

Questions, Answers, Explanations, and Connections

Part A

Parking Lot

Story There are 12 spaces for cars in a parking lot. The pictures below show which spaces were used on Tuesday. Parking lot on Monday Parking lot on Monday Parking lot on Tuesday Image: Colspan="2">Image: Colspan="2">Image: F Image: Colspan="2">Image: Colspan="2">Image: F Image: Colspan="2">Image: Colspan="2">Image: F Image: Colspan="2">Image: Colspan="2">Image: F Image: Colspan="2">Image: Colspan="2" Image: Colspan="2

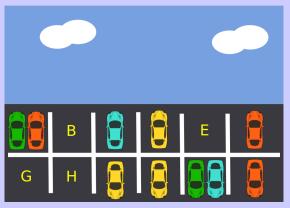
Question	
How many parking spaces were empty on both Monday and Tuesday?	
(A) 3	
(B) 4	
(C) 5	
(D) 6	

(B) 4

Explanation of Answer

We can see which spaces were used by placing the cars from both days on the parking lot at the same time.

Parking lot on Monday and Tuesday



Then we can simply count empty spaces to determine that four spaces were empty on both Monday and Tuesday.

Connections to Computer Science

All *data* can be thought of as a sequence of zeros and ones. Each zero or one is called a *bit* and the sequence is called a *binary code*, *binary representation*, or *binary number*.

In this task, we can model an empty space as 0 and a space with a car as 1; each parking space corresponds to a bit. We get a sequence of bits by viewing the parking spaces in order. For example, we can read the top row from left to right and then the bottom row from left to right to get 101001001010 from the parking lot on Monday and 100100000111 from the parking lot on Tuesday. This task asks to determine which of the twelve positions contain a 1 in either of these binary numbers. This is a logical operation named OR. Notice how we can compute the correct answer by seeing that

 101001001010

 OR
 100100000111

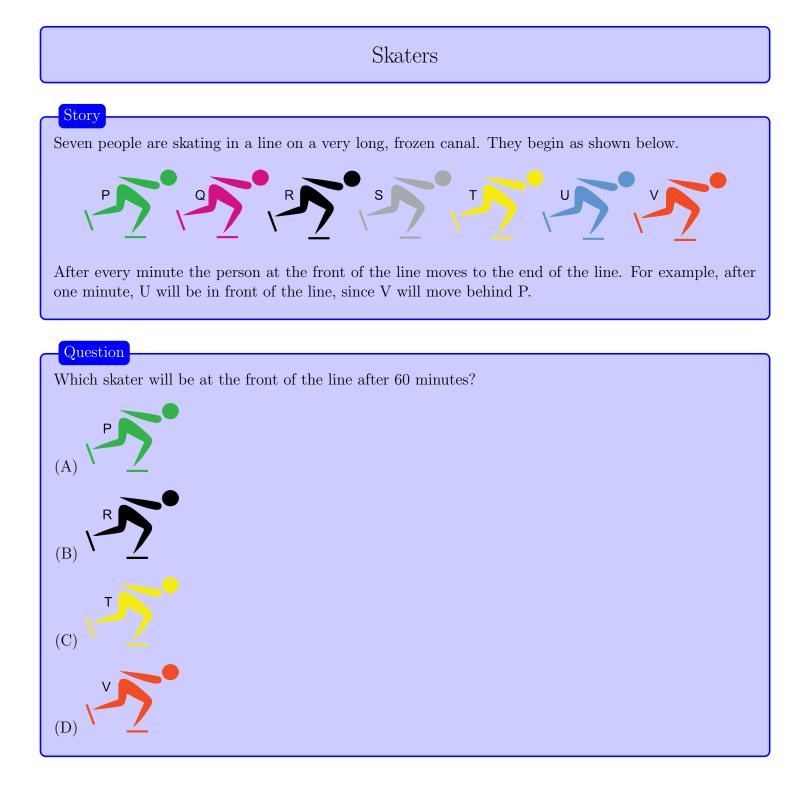
 1011010011111

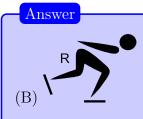
This resulting binary number has four zeros in it.

Country of Original Author

Canada







Explanation of Answer

There are seven skaters. So after every seven minutes, the skater originally at the front of the line returns to the front of the line. Now note that there are 60 minutes in an hour and $60 = 56 + 4 = 7 \times 8 + 4$. So after 56 minutes, the line of skaters is the same as at the beginning. Four minutes later (after one hour), skaters V, U, T and S will have moved to the end of the line (in that order). Therefore, after 60 minutes, skater R will be at the front of the line.

Connections to Computer Science

The solution to this problem involved finding the quotient and remainder when dividing 60 by 7. Computing and working with remainders is called *modular arithmetic*. This mathematics is central to many areas of computer science. Modular arithmetic is related to how we *represent integers* in computer memory. It also plays a central role in *cryptography* which is the study of sending and receiving secret message.

A third use of modular arithmetic arises when arranging and manipulating data similar to the skaters in this problem. We can abstractly view a line of data by what computer scientists call a *queue*. One way to store the data in a queue in memory, is to use a *"circular" array-based queue*. This data structure allows data to be efficiently removed from the front of the queue and added to the rear of the queue. Modular arithmetic is usually used in the details of the implementation.

Country of Original Author

Canada



Risk

Story

Darren's computer is connected to the Internet but does not have any antivirus or firewall software. None of the accounts on his computer are protected by a password.



Question

Which computers are at risk because of this?

- (A) only Darren's own computer
- (B) only the computers in the same room as Darren's computer
- (C) only the computers in the same country as Darren
- (D) all computers in the world which are connected to the Internet and set up like Darren's

Answer

(D) all computers in the world which are connected to the Internet and set up like Darren's

Explanation of Answer

Due to Darren's reckless behaviour, his own computer may get infected by malicious software. However, infected computers are often used as a platform for attacking other computers. This means that all computers connected to the Internet without proper protection, such as firewalls and antivirus software are at risk because of Darren's actions.

Connections to Computer Science

Owners of other computers can reduce some of the risks by following good practices. For example, installing *security updates* and running *antivirus software* reduces the risk of malware spreading to other computers. Unfortunately, this does not eliminate all risks. *Malware*, which is software designed to do malicious things, if it is on Darren's computer, may be used to send spam or overload network connections (these are called *DDoS attacks*), and there is very little other computer users can do to protect themselves against these larger scale attacks.

Cybersecurity (or *Information Technology security*) helps protect computer systems from theft or damage to the hardware, software or the information on them, as well as from disruption or misdirection of the services they provide.

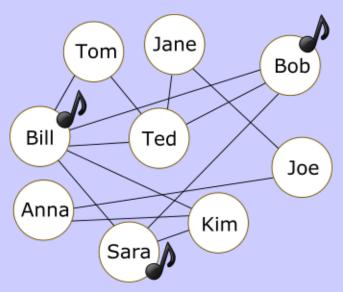
Country of Original Author

Estonia

Popular Song

Story

The lines in the diagram show exactly which pairs of students in a class are friends. A popular artist releases a new song on Monday and there is a musical note beside each student that buys the song that day.



Every day after that, if a student has not bought the song yet but at least half of their friends did buy the song before this day, he or she will also buy the song. Otherwise they do not buy the song yet.

Question

What is the earliest day when all students in the class own the song?

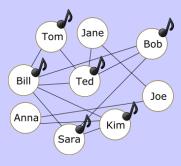
- (A) Wednesday
- (B) Thursday
- (C) Saturday
- (D) Sunday

Answer

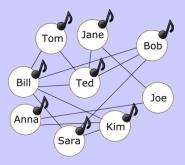
(B) Thursday



Tom, Ted and Kim buy the song on Tuesday:



Anna and Jane buy the song on Wednesday:



Finally Joe will buy the song on Thursday.

Connections to Computer Science

This task models a *social network* of friends by way of a *graph*. Each person is a *vertex* in the graph, and two friends are connected by an *edge*. The task is based on determining the *spread of influence through a social network*. This spread of influence can be modelled by defining a *threshold of diffusion*: that is, every person changes his/her mind if a certain ratio of his/her neighbours have a different idea.

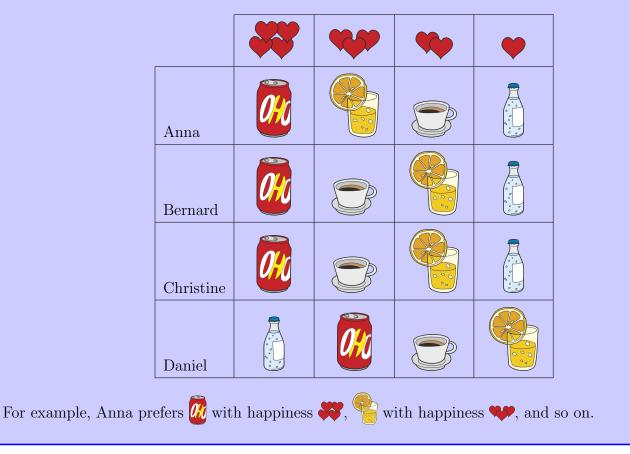
Social media companies and marketing companies extract information about connections between individuals and purchasing preferences in order to attempt to market new products or ideas to individuals, based on the proportion of friends who have purchased those products or agree with the particular idea.



Soda Shoppe

Story

Four friends each buy one drink. How happy each person will be for each type of drink is shown in the table below. The more hearts indicated, the happier a person will be. Unfortunately, only one of each of the four types of drinks is available.



Question

When measured by the total number of hearts, what is the happiest the friends can be?

- (A) 13
- (B) 14
- (C) 15
- (D) 16

(B) 14

Since three people want \mathcal{U} , the maximum total number of hearts the group could get is 4+4+3+3 = 14. This is obtainable via the following matching:



Another way to see this, since Daniel has () as his favourite drink and all others rank this beverage last, Daniel must pick () to maximize the group's happiness. Now, the first three people have the same first preference however the Anna has () as the next favourite and the other two people have this as the third favourite. Thus Anna should pick (). The other two people can pick () and () in whichever order.

Connections to Computer Science

Optimization is a very important part of computer science. In this situation, we are trying to maximize the happiness of the group. Optimization problems are ubiquitous in computer science.

This problem is a specific instance of an *optimal matching problem*. In this task, friends are trying to get matched to the best possible drink to make the group as happy as possible. These types of problems are extremely important in the real world. As an example, patients who are waiting for an organ transplant are place on a huge list. However, not all organs can be used on a person – organ donors must, for example, match their blood types in order to have a chance at a successful operation. These constraints make it harder to find a successful matching.

Country of Original Author

Canada



Part B

Balls

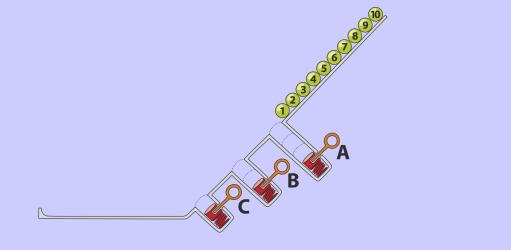
Story

Numbered balls roll down a ramp as shown below. When a ball comes to a hole, if there is enough space, the ball falls in. Otherwise, the ball rolls past the hole. A pin at the bottom of each hole can be pulled which ejects the balls.



Note the final ordering of the balls in the example above is 45321.

In the diagram below, ten balls roll down the ramp. Three holes, with pins labelled A, B, and C, have space for 3, 2 and 1 balls, respectively. After all ten balls stop moving, pin A is pulled. Then, after all the balls released from A stop moving, pin B is pulled. Finally, after all the balls released from B stop moving, pin C is pulled.



Question

Which of the following is the final ordering of the balls?

- (A) **78910123456**
- (B) **78910123546**
- (C) **10987654321**
- (D) **78910321546**

(D) **78910321546**

Explanation of Answer

Hole A has space for three balls, so balls 4 to 10 roll past it in order. Hole B has space for two balls, so balls 6 to 10 roll past it in order. Hole C has space for one ball, so balls 7 to 10 roll past it in order. Then the pin in hole A is pulled and balls are ejected in the order 3,2,1 and roll to the bottom. At this point, the balls are at the bottom in the order 7,8,9,10,3,2,1. Then the pin in hole B is pulled and balls are ejected in the bottom in the order 7,8,9,10,3,2,1. Then the pin in hole B is pulled and balls are ejected in the order 5,4. At this point, the balls are at the bottom in the order 7,8,9,10,3,2,1,5,4. Finally, the pin is pulled in hole C and ball 6 rolls to the bottom giving the correct answer.

Connections to Computer Science

The holes act like a *stack* which is a *data structure*. A data structure is a way of organizing data to make certain operations, like *insert*, *delete*, or *retrieve*, more efficient. A stack is based on the *last-in first-out principle (LIFO)*. For example, the first ball to fall into a hole, is the last ball ejected from the hole. Despite being a very simple idea, it is useful in many different situations.

As an example, a stack can be used to check that the brackets in a mathematical expression like ((1+2)*3) are *balanced* but the brackets in ((4+5)*(6-7) are not balanced. The way to perform this check is that all opening brackets should be put on a stack (this operation is called a *push*) and when its matching closing bracket is found, the opening bracket is removed from the top of the stack (this operation is called a *pop*).

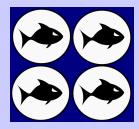
Country of Original Author

Serbia



Story

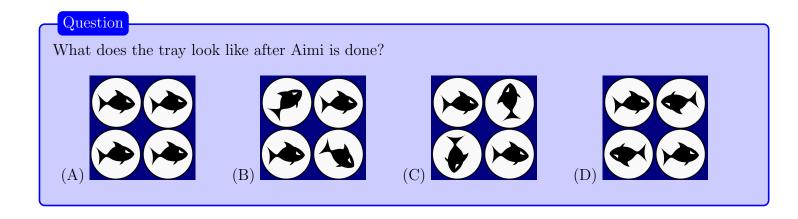
Four fish are placed on a tray as shown below.



Every time a fish is turned clockwise, the fish diagonal to it also turns the same amount but in a counter-clockwise direction. For example, if the fish in the upper right turns 90° clockwise, then the fish in the lower left turns 90° counter-clockwise.

Aimi makes the following four turns in order.

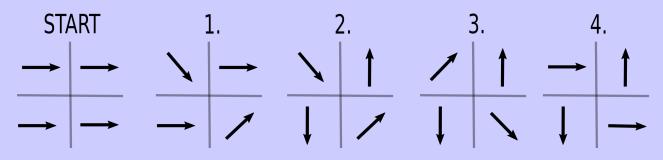
- 1. Turn the fish in the upper left 45° clockwise.
- 2. Turn the fish in the lower left 90° clockwise.
- 3. Turn the fish in the lower right 90° clockwise.
- 4. Turn the fish in the upper left 45° clockwise.



Answer (C)

Explanation of Answer

A student who tries to follow this in their head is likely to get confused. A simpler way of solving this task reliably is required. One way is to produce a quick and simple notation to keep track of the changes. The system shown below is clear and easy to produce if arrows are used instead of fish.



Alternatively this task can be quickly solved by reading all the operations and noticing that only step number 2 affects toys on the lower-left and upper-right. Therefore a student can simply look at the answer options and choose the only one with lower-left and upper-right fish in the correct position.

Connections to Computer Science

One way to solve this problem is remembering the *state* of each fish, in terms of its position after each turn. To determine the correctness of a program, typically we need to determine the state of all variables at each step of the program. This process of *debugging* or *tracing* of a program is an important part of designing and reasoning about the correctness of computer programs.

Computers, in particular the *central processing unit* or CPU, can be modelled as a *finite state* machine which keeps track of its current state and will change its state based on input. This problem can be simulated with a finite state machine with 4 states (one for each corner of the digram), and transitions for each possible rotation of each fish to its corresponding next state.

Country of Original Author

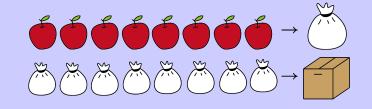
Japan

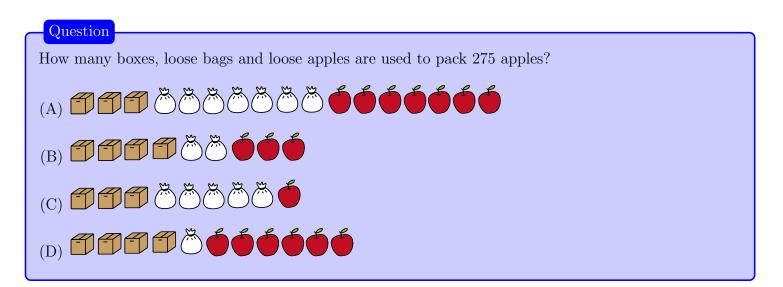
Apple Packing

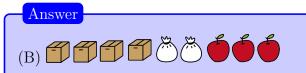
Story

At the Beaver Apple Orchard, apples are either sold individually, or in bags of 8 apples (called Family Packs), or in boxes of 8 Family Packs. Therefore, apples are packed according to the following rules:

- 1. Apples are put in bags. Each bag can only hold exactly 8 apples. If there are less than 8 apples, they remain outside the bags as loose apples.
- 2. Bags are put in boxes. Each box can hold exactly 8 bags. If there are less than 8 bags, they are left outside the boxes as loose bags.







Explanation of Answer

Each box contains a total of 64 apples (8 bags \times 8 apples per bag). The beavers picked 275 apples which filled four boxes (275 ÷ 64 = 4.296875). These four boxes contains a total of 64 \times 4 = 256 apples. Since each bag holds 8 apples, the remaining 275 - 256 = 19 apples filled 2 loose bags (19 ÷ 8 = 2.375). These two loose bags contain 16 apples leaving 3 loose apples left.

As a check:

Choice A has a total of $3 \times 64 + 7 \times 8 + 7 = 255$ apples Choice B has $4 \times 64 + 2 \times 8 + 3 = 275$ apples Choice C has $3 \times 64 + 5 \times 8 + 1 = 233$ apples Choice D has $4 \times 64 + 1 \times 8 + 6 = 270$ apples

Connections to Computer Science

The *binary number system* is the simplest form of computer code or information. The binary number system uses *binary numbers*, namely a sequence of zeros and ones, to represent all information.

The octal system, on the other hand, uses eight different symbols, values from 0 to 7. If any value larger than 7 is needed, extra columns are added to the left. Each value in one column is 8 times greater than the value in the column to its immediate right. The octal system allows the same information to be represented using only a third of the digits as each octal digit represents three binary digits. This was ideal for some old computer systems as they could concisely display certain machine code.

In the question, the boxes represent the number of $8^2 = 64$ apples, the loose bags represent the number of $8^1 = 8$ apples, and the loose apples represent the number of $8^0 = 1$ (units). Thus, the decimal (base-10) number 275 is expressed as 423 in the octal (base-8) system.

Country of Original Author

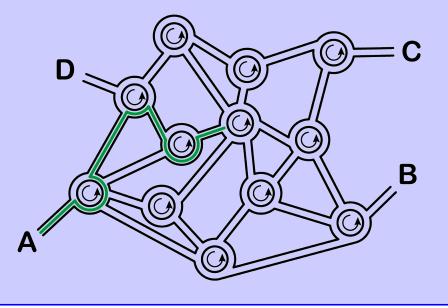
Taiwan



Roundabout City

Story

In Roundabout City, navigation software gives instructions as a sequence of numbers, representing which exit number to take at each roundabout. For example, the instructions "4 1 2" mean to take the 4th exit at the first roundabout, the 1st exit at the next roundabout, and the 2nd exit at the next roundabout. The diagram shows this route highlighted in green, beginning at A.



Question

If we start from A and follow the sequence "3 1 3 2 3" where will we end up?

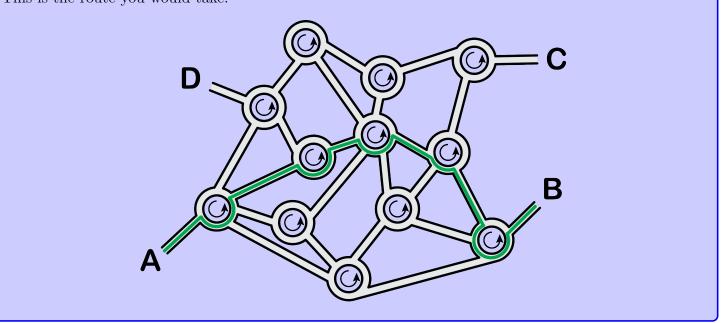
(A) A

(B) B

(C) C

(D) D

This is the route you would take:



Connections to Computer Science

This task introduces one important element of many computer programming languages: the instructions in a computer program are performed *sequential*, one after another in order. Additionally, each individual instruction should be performed *exactly as specified*.

The sequence of instructions given in the task, which is the exit number at each roundabout, can be thought of as a program in a very simple programming language. In order to achieve the desired goal or *output*, each instruction must be followed exactly (e.g., the sequence "4 1 2" is different than "3 1 2") and in the order specified (e.g., the sequence "4 1 2" is not the same as the sequence "2 4 1").

Country of Original Author

Slovenia

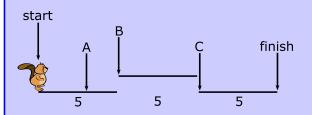


Jumpers

Story

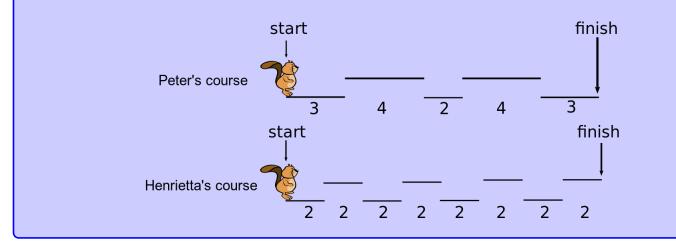
Peter and Henrietta are playing a video game. They move a beaver at a constant speed from the start of a course to the finish. The course consists of platforms on two levels. At the end of each platform before the finish, the beaver jumps instantaneously up or down to the next platform. The amount of time to move over each platform of the game is shown below each platform.

Here is an example course:



- 3 seconds after the start, the beaver is at A;
- 5 seconds after the start, the beaver is at B;
- 10 seconds after the start, the beaver is at C;
- 15 seconds after the start, the beaver is at the finish.

Peter and Henrietta start playing the following two different courses at exactly the same time.



Question

For how long are both beavers moving along the top level at the same time?

- (A) 2 seconds
- (B) 4 seconds
- (C) 6 seconds
- (D) 8 seconds

(B) 4 seconds

Explanation of Answer

We can see this answer by writing down "being at the bottom level for one second" as a 0, and "being at the top level for one second" as 1. Then, Peter's game can be described as:

0001111001111000

and Henrietta's game can be described as:

0011001100110011

To find the times that they are both on the upper level, we need to find the times when both games have a 1 value simultaneously. This can be done by applying the boolean logic function AND between these two sequences to get:

Connections to Computer Science

One of the main philosophies in computer science is how to represent information. In this task, we can *abstract* (or hide) away some of the details of the game to focus on what matters, which in this case is when a player is on the upper level. Then, once the *information* is represented in a precise way, we can *transform* or *combine* the information into new and meaningful ways. Specifically, for this problem, we treat the information as sequences of *binary numbers* and perform a *bit-wise* AND operation to find where both sequences have a 1 value simultaneously.

Country of Original Author

Canada



Part C

Intrusion

Story

In the Bebras Museum of Post-Modern Wood Art, there is an intelligent security system that detects intruders. An intruder is a person who has entered the museum, but not via the entrance. The entrance of the museum is in Room 1.

Each and every time a person enters or leaves a room, the system detects exactly how many people are in each room and records this in a table. The system always correctly allocates each person in the museum to a single room. It may happen that several people enter or leave a room at the same time.

The table shows the records of the intelligent security system and the image shows the layout of the rooms in the museum.

Time	Room 1	Room 2	Room 3	Room 4		
10:00	2	0	0	0	1	2
10:07	3	0	0	0		
10:08	2	1	0	0	\vdash —	\vdash \dashv
10:12	4	1	1	0		
10:13	2	2	3	0	4	3
10:17	5	2	2	1		Ň
10:20	4	1	2	2		
10:20	4	1	Z	Z		

Question

At what time did the system detect an intruder?

(A) 10:07

- (B) 10:12
- (C) 10:13
- (D) 10:17

(C) 10:13

Explanation of Answer

At 10:13 two people entered room 3, but in the attached rooms there was only one person before (in room 2). Thus someone entered room 3 from outside the museum not using the entrance.

Connections to Computer Science

Security systems keeping track of the number of people is critical in places such as airports, train stations or government buildings. Computer programs can evaluate and extract information from images, including the ability to detect people and thus count them. These programs use *artificial intelligence*, for example, to recognize humans, but also use simple *logical rules* like those outlined in this task (i.e., the number of people in a room cannot be more than the sum of all people in the room before plus the number of people in adjoining rooms) in order to detect security breaches.

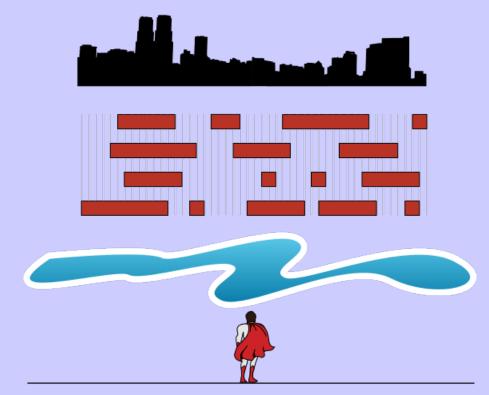
Country of Original Author

Germany

Super Hero

Story

A super hero watches over Beaver City from a straight path across a river. From every point along the path, the super hero needs to be able to see the point in the city directly across the river. Unfortunately, 16 walls of varying lengths stand between the river and the city as shown.



Fortunately, the super hero has X-ray vision and can see through a wall.

Unfortunately, the super hero can only see through one wall at a time.

Fortunately, the super hero is strong enough to destroy walls, and when he destroys a wall, he destroys it completely.

Unfortunately, destroying a wall makes the super hero very tired.

Question What is the fewest number of walls that the super hero needs to destroy?
(A) 9
(B) 10
(C) 11
(D) 12

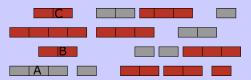
٢	Answer			
L	(A) 9			
	、 <i>'</i>			

Explanation of Answer

Instead of determining the fewest number of walls that the super hero needs to destroy, we consider the largest number of walls that the super hero can keep (not destroy). This is equivalent.

We will show that the largest number of walls that the super hero can keep is seven. Since there are at total of 16 walls, this means the super hero needs to destroy 16 - 7 = 9 walls.

Consider the seven grey walls in the picture below.



No two of these seven walls "overlap". That is, from every point along the path, at most one of these walls will be between the super hero and the point in the city directly across the river. Thus we know that the super hero can keep at least seven walls. We must show that the super hero cannot keep more than seven walls.

There are walls of four lengths. Let's name these lengths 1, 2, 3 and 4 (which happen to be proportional to the actual lengths in the picture).

Interestingly, instead of the leftmost wall "A" of length 3 in our solution, the super hero could keep either of the two leftmost walls of length 2, "B" or "C". So there is an alternative set of seven walls of lengths 2, 1, 2, 1, 1, 2 and 1 that the super hero could keep. The super hero cannot do better than this because there are only five walls of length 1 and two of them "overlap".

Why did we choose a wall of length 3 in our solution? We did this because it comes from a way to solve this problem for any set and placement of walls, not just those in this particular task. More generally, we can find the walls to keep by starting from the left and always selecting the next (non-overlapping) wall that ends closest to the left. (Can you see why this always works?)

Another way to observe that we can't destroy less than 9 walls is to notice that there are disjoint sets of overlapping walls of sizes 4, 3, 4, and 2 when reading from left to right. This means that at least 3 + 2 + 3 + 1 = 9 walls must be destroyed.

Connections to Computer Science

The general solution to this problem is a *greedy algorithm*. That is, it is a sequence that solves a problem by always making a choice at each step that seems best "locally" or "at the moment". It does not take into account the "big picture" or full set of information.

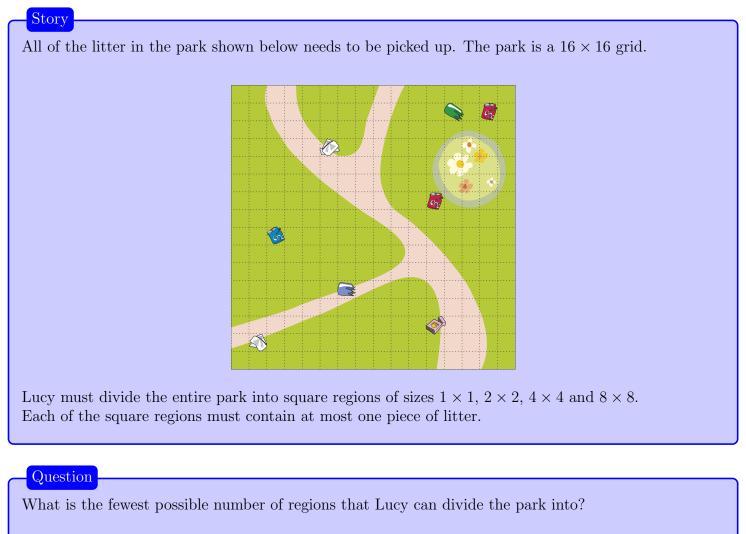
This particular problem is a model of a *scheduling problem* for which computer scientists are interested in solving with *efficient algorithms*. To see this, think of time moving forward as you move towards from left to right and think of the walls as tasks that need to be completed with a fixed start and end time. The restriction that the super hero can only see through one wall corresponds to the fact that only one task can be completed at any one time. In our problem, the super hero is trying to solve as many tasks as possible!

Country of Original Author

Canada



Litter



(A) 9

(B) 13

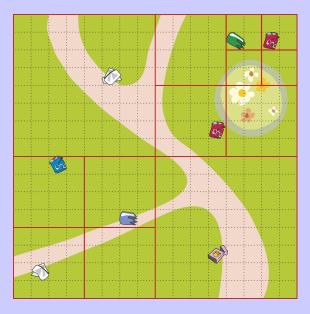
(C) 16

(D) 64

(B) 13

Explanation of Answer

The solution is shown below, where the red lines indicate where we divide the park into regions:



Notice that we must use at least 8 square regions, since there are 8 pieces of litter. To get the minimum number of squares, we want to maximum the size of each square region around each piece of litter. For the solution shown, there is no larger square of the given sizes that can surround each piece of litter without overlapping another piece of litter.

Connections to Computer Science

This representation of two-dimensional information is called a *quad-tree*. A quad-tree splits a twodimensional region into four equal parts (*quad* meaning four in Latin), where each part may be further subdivided into four equal parts, until there is only one item in each part. The use of quad-trees as a *data structure* allows graphical information (such as the positions of windows on a computer screen) to be stored and searched efficiently, in particular, allowing *range queries* of the form "how many objects are in this part of a picture?" to be performed in a very efficient manner.

Country of Original Author

Canada

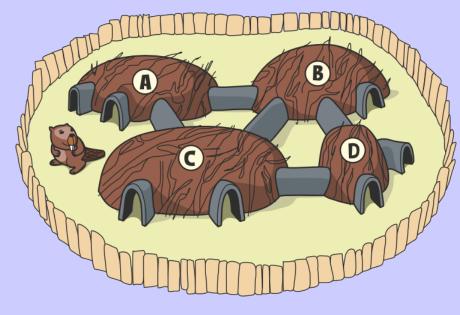


Beaver Lodge

Story

The Beaver family has built a lodge with 4 rooms and 5 tunnels connecting rooms as shown. There are also 7 doorways to the outside.

The Beaver children have noticed that it is possible to start in one of the rooms and run along a path passing through all of the tunnels and all of the doorways without walking through any doorway or tunnel twice.



Question

In which room did the Beaver children start running along such a path?

- (A) Room A
- (B) Room B
- (C) Room C
- (D) Room D

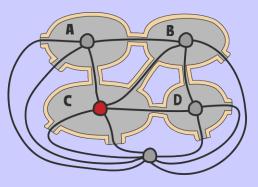
(C) Room C

Explanation of Answer

Turn each room into a node with an additional node for the outside. The nodes are connected if and only if there is a tunnel between the rooms presented by them or a doorway if one of the nodes represents the outside.

The described path is possible if there are no more than two nodes with an odd number of edges. Such nodes will be the first and last node on a path.

In our case the starting node with an odd number of edges (3 tunnels + 2 doorways) has to be C.



Connections to Computer Science

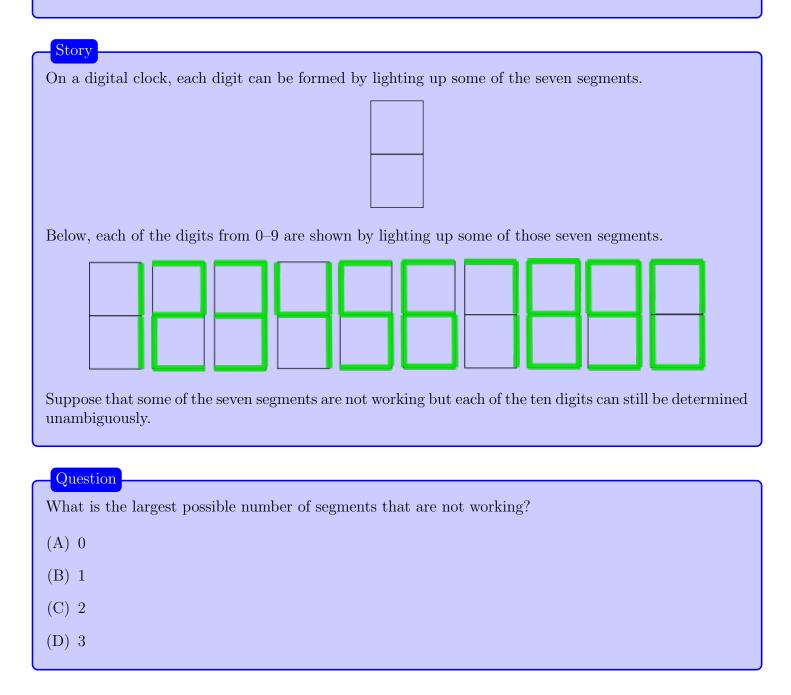
We can transform this task into a *graph theory* problem, where each location (room or outside) is represented as a *node* and each tunnel or doorway is represented as one *edge*. The path which this task asks to find is called an *Eulerian path*: that is, a path in a graph which visits every node exactly once. In order for there to be an Eulerian path in a graph, it is necessary that either zero or two nodes have an odd *degree*, where the degree of a node is the number of edges connected to that node. To see why that condition is necessary, notice that a continuous path will leave each node exactly as often as enters it, except for nodes which are at the start and at the finish.

Many problems are modelled using graphs, and computers algorithms for *shortest paths*, *cycle finding*, *connectivity*, and *strongly connected components* are some examples of well-known and well-studied algorithms that have been developed to solve real-world problems involving graphs.

Country of Original Author

Lithuania

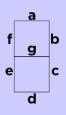
Broken Digital Clock



(C) 2

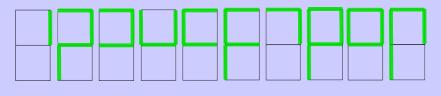
Explanation of Answer

Consider the following labelling of the 7 segments of one digit, with the letters a through g:



Segment a must work to distinguish between digits 1 and 7. Segment b must work to distinguish between digits 6 and 8. Segment e must work to distinguish between digits 5 and 6. Segment f must work to distinguish between digits 3 and 9. Segment g must work to distinguish between digits 0 and 8.

If both segments c and d were not working, we would have the following 10 digits, each of which is uniquely determined:



Connections to Computer Science

Pattern recognition algorithms are algorithms with take in complex information, such as a picture or sound, and try to categorize it. For example, pattern recognition algorithms are used to detect facial features, such as eyes, mouth, and nose, for security and social media applications. These algorithms look for *identifying information* and attempt to provide a "most likely" answer, using statistical models.

Pattern recognition is a branch of a larger area of computer science called *machine learning*, that focuses on the recognition of patterns and regularities in data. One of the approaches to recognition is to extract specific features, that allow uniquely identify objects. This task focuses on identifying these key features to distinguish between each digit.

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