



The CENTRE for EDUCATION  
in MATHEMATICS and COMPUTING  
*cemc.uwaterloo.ca*

# *Pascal Contest*

*(Grade 9)*

**Wednesday, February 24, 2016**  
*(in North America and South America)*

**Thursday, February 25, 2016**  
*(outside of North America and South America)*



UNIVERSITY OF  
**WATERLOO**

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**Time:** 60 minutes

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Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.

## **Instructions**

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D, and E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.
10. You may not write more than one of the Pascal, Cayley or Fermat Contest in any given year.

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*Do not discuss the problems or solutions from this contest online for the next 48 hours.*

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*The name, grade, school and location, and score range of some top-scoring students will be published on our website, [cemc.uwaterloo.ca](http://cemc.uwaterloo.ca). In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.*

Scoring: There is *no penalty* for an incorrect answer.  
 Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

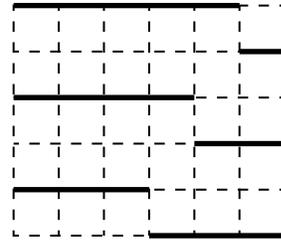
**Part A: Each correct answer is worth 5.**

1. The result of the addition shown is  
 (A) 15021      (B) 12231      (C) 12051  
 (D) 13231      (E) 12321

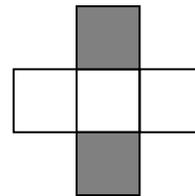
$$\begin{array}{r} 300 \\ 2020 \\ + 10001 \\ \hline \end{array}$$

2. Which of the following has the largest value?  
 (A)  $4^2$       (B)  $4 \times 2$       (C)  $4 - 2$       (D)  $\frac{4}{2}$       (E)  $4 + 2$

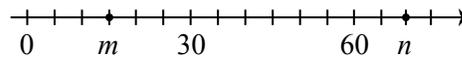
3. In the diagram, the  $5 \times 6$  grid is made out of thirty  $1 \times 1$  squares. What is the total length of the six solid line segments shown?  
 (A) 6      (B) 12      (C) 16  
 (D) 18      (E) 20



4. In the diagram, each of the five squares is  $1 \times 1$ . What percentage of the total area of the five squares is shaded?  
 (A) 25%      (B) 30%      (C) 35%  
 (D) 40%      (E) 45%

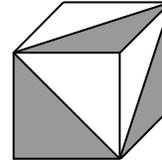


5. Numbers  $m$  and  $n$  are on the number line, as shown. The value of  $n - m$  is  
 (A) 66      (B) 35      (C) 55  
 (D) 60      (E) 54



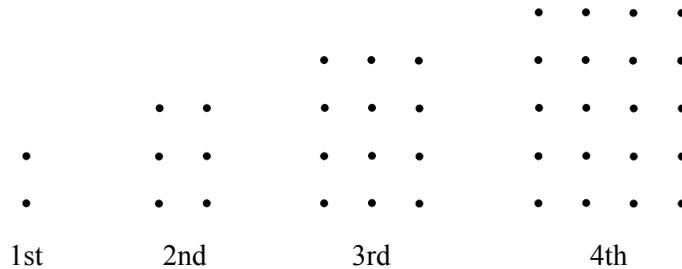
6. If the symbol  $\frac{p}{r} \left| \frac{q}{s} \right.$  is defined by  $p \times s - q \times r$ , then the value of  $\frac{4}{2} \left| \frac{5}{3} \right.$  is  
 (A) -3      (B) -2      (C) 2      (D) 3      (E) 14
7. Which of the following is equal to 2 m plus 3 cm plus 5 mm?  
 (A) 2.035 m      (B) 2.35 m      (C) 2.0305 m      (D) 2.53 m      (E) 2.053 m
8. If  $x = 3$ ,  $y = 2x$ , and  $z = 3y$ , then the average of  $x$ ,  $y$  and  $z$  is  
 (A) 6      (B) 7      (C) 8      (D) 9      (E) 10

9. A soccer team played three games. Each game ended in a win, loss, or tie. (If a game finishes with both teams having scored the same number of goals, the game ends in a tie.) In total, the team scored more goals than were scored against them. Which of the following combinations of outcomes is not possible for this team?
- (A) 2 wins, 0 losses, 1 tie  
 (B) 1 win, 2 losses, 0 ties  
 (C) 0 wins, 1 loss, 2 ties  
 (D) 1 win, 1 loss, 1 tie  
 (E) 1 win, 0 losses, 2 ties
10. Exactly three faces of a  $2 \times 2 \times 2$  cube are partially shaded, as shown. (Each of the three faces not shown in the diagram is not shaded.) What fraction of the total surface area of the cube is shaded?
- (A)  $\frac{1}{3}$                       (B)  $\frac{1}{4}$                       (C)  $\frac{1}{6}$   
 (D)  $\frac{3}{8}$                       (E)  $\frac{2}{3}$



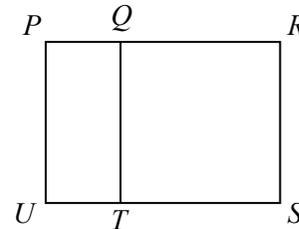
**Part B: Each correct answer is worth 6.**

11. An *oblong number* is the number of dots in a rectangular grid with one more row than column. The first four oblong numbers are 2, 6, 12, and 20, and are represented below:



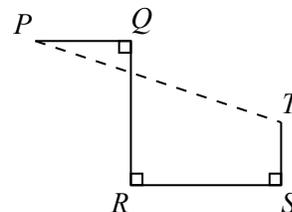
What is the 7th oblong number?

- (A) 42                      (B) 49                      (C) 56                      (D) 64                      (E) 72
12. In the diagram, the area of square  $QRST$  is 36. Also, the length of  $PQ$  is one-half of the length of  $QR$ . What is the perimeter of rectangle  $PRSU$ ?



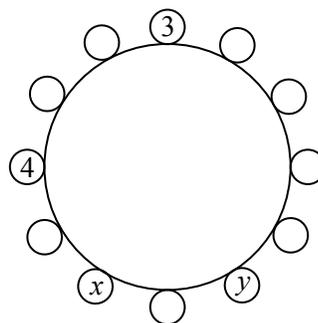
- (A) 24                      (B) 30                      (C) 90  
 (D) 45                      (E) 48
13. Multiplying  $x$  by 10 gives the same result as adding 20 to  $x$ . The value of  $x$  is
- (A)  $\frac{9}{20}$                       (B)  $\frac{20}{9}$                       (C)  $\frac{11}{20}$                       (D)  $\frac{20}{11}$                       (E) 2

14. In the diagram,  $PQ$  is perpendicular to  $QR$ ,  $QR$  is perpendicular to  $RS$ , and  $RS$  is perpendicular to  $ST$ . If  $PQ = 4$ ,  $QR = 8$ ,  $RS = 8$ , and  $ST = 3$ , then the distance from  $P$  to  $T$  is
- (A) 16                      (B) 12                      (C) 17  
 (D) 15                      (E) 13



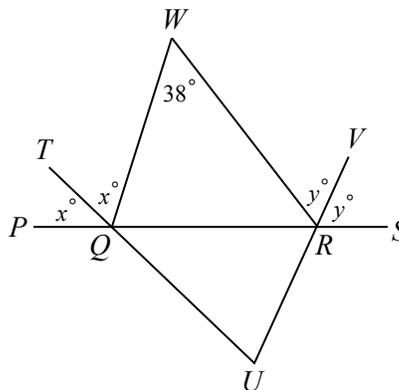
15. When two positive integers  $p$  and  $q$  are multiplied together, their product is 75. The sum of all of the possible values of  $p$  is  
 (A) 96            (B) 48            (C) 109            (D) 115            (E) 124
16. An integer from 10 to 99 inclusive is randomly chosen so that each such integer is equally likely to be chosen. The probability that at least one digit of the chosen integer is a 6 is  
 (A)  $\frac{1}{5}$             (B)  $\frac{1}{10}$             (C)  $\frac{1}{9}$             (D)  $\frac{19}{90}$             (E)  $\frac{19}{89}$
17. What is the tens digit of the smallest six-digit positive integer that is divisible by each of 10, 11, 12, 13, 14, and 15?  
 (A) 0            (B) 6            (C) 2            (D) 8            (E) 4

18. Each integer from 1 to 12 is to be placed around the outside of a circle so that the positive difference between any two integers next to each other is at most 2. The integers 3, 4,  $x$ , and  $y$  are placed as shown. What is the value of  $x + y$ ?  
 (A) 17            (B) 18            (C) 19  
 (D) 20            (E) 21



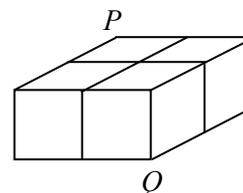
19. Chris received a mark of 50% on a recent test. Chris answered 13 of the first 20 questions correctly. Chris also answered 25% of the remaining questions on the test correctly. If each question on the test was worth one mark, how many questions in total were on the test?  
 (A) 23            (B) 38            (C) 32            (D) 24            (E) 40

20. In the diagram, points  $Q$  and  $R$  lie on  $PS$  and  $\angle QWR = 38^\circ$ . If  $\angle TQP = \angle TQW = x^\circ$ ,  $\angle VRS = \angle VRW = y^\circ$ , and  $U$  is the point of intersection of  $TQ$  extended and  $VR$  extended, then the measure of  $\angle QUR$  is  
 (A)  $71^\circ$             (B)  $45^\circ$             (C)  $76^\circ$   
 (D)  $81^\circ$             (E)  $60^\circ$



**Part C: Each correct answer is worth 8.**

21. Grid lines are drawn on three faces of a rectangular prism, as shown. A squirrel walks from  $P$  to  $Q$  along the edges and grid lines in such a way that she is always getting closer to  $Q$  and farther away from  $P$ . How many different paths from  $P$  to  $Q$  can the squirrel take?  
 (A) 14            (B) 10            (C) 20  
 (D) 12            (E) 16

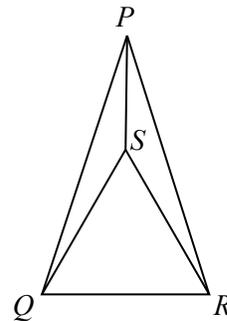


22. There are  $n$  students in the math club at Scoins Secondary School. When Mrs. Fryer tries to put the  $n$  students in groups of 4, there is one group with fewer than 4 students, but all of the other groups are complete. When she tries to put the  $n$  students in groups of 3, there are 3 more complete groups than there were with groups of 4, and there is again exactly one group that is not complete. When she tries to put the  $n$  students in groups of 2, there are 5 more complete groups than there were with groups of 3, and there is again exactly one group that is not complete. The sum of the digits of the integer equal to  $n^2 - n$  is

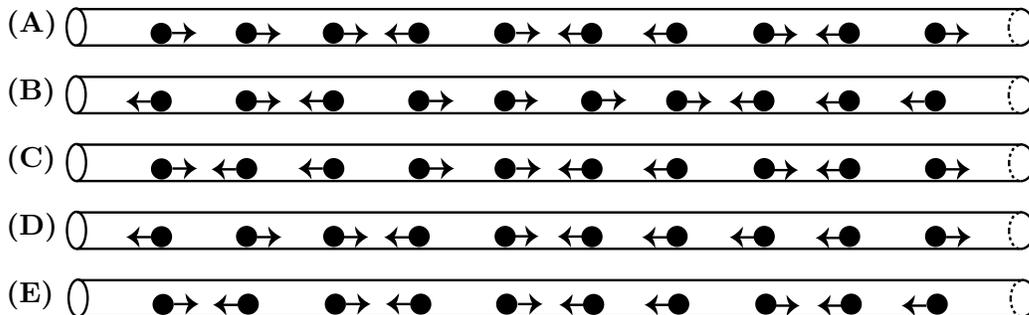
(A) 11            (B) 12            (C) 20            (D) 13            (E) 10

23. In the diagram,  $\triangle PQR$  is isosceles with  $PQ = PR = 39$  and  $\triangle SQR$  is equilateral with side length 30. The area of  $\triangle PQS$  is closest to

(A) 68            (B) 75            (C) 50  
(D) 180            (E) 135

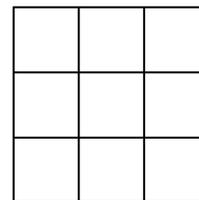


24. Ten very small rubber balls begin equally spaced inside a 55 m long tube. They instantly begin to roll inside the tube at a constant velocity of 1 m/s. When a ball reaches an end of the tube, it falls out of the tube. When two balls bump into each other, they both instantly reverse directions but continue to roll at 1 m/s. Five configurations giving the initial direction of movement of each ball are shown. All gaps indicated in the diagram are the same length and are equal in length to the distance from the ends of the tube to the nearest ball. For which configuration will it take the least time for more than half of the balls to fall out of the tube?



25. A 0 or 1 is to be placed in each of the nine  $1 \times 1$  squares in the  $3 \times 3$  grid shown so that each row contains at least one 0 and at least one 1, and each column contains at least one 0 and at least one 1. The number of ways in which this can be done is

(A) 126            (B) 120            (C) 138  
(D) 102            (E) 96





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**For students...**

Thank you for writing the 2016 Pascal Contest! Each year, more than 220 000 students from more than 60 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Fryer Contest which will be written in April.

Visit our website [cemc.uwaterloo.ca](http://cemc.uwaterloo.ca) to find

- More information about the Fryer Contest
- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

**For teachers...**

Visit our website [cemc.uwaterloo.ca](http://cemc.uwaterloo.ca) to

- Register your students for the Fryer, Galois and Hypatia Contests which will be written in April
- Look at our free online courseware for senior high school students
- Learn about our face-to-face workshops and our web resources
- Subscribe to our free Problem of the Week
- Investigate our online Master of Mathematics for Teachers
- Find your school's contest results