WATERLOO | MATHEMATICS The CENTRE for EDUCATION in MATHEMATICS and COMPUTING





Questions

Part A

Boxes



Question

How should she fill out the form to order boxes totaling exactly 20 kilograms?



Propagate

Story

Roberta Beaver has purchased an old computer that only allows one digit after the decimal point in any calculation. Anything after that digit is removed. Sometimes this results in an *error* which is the difference between the stored value and the exact value.

For example, if we try to compute $\frac{7}{5}$ on Roberta's machine, this will be stored as 1.4 which is the exact value of $\frac{7}{5}$. This gives an error of 0. However, if we compute $\frac{7}{4}$, this will be stored as 1.7 since $\frac{7}{4} = 1.75$ and "5" will be removed from the end. This gives an error of 0.05.

Extra digits are removed after every operation. For example, when Roberta computes $(\frac{3}{2}) \times (\frac{2}{3})$, she computes $\frac{3}{2}$ to give 1.5, then $\frac{2}{3}$ to give 0.6, and then 1.5×0.6 to give 0.9. This gives an error of 0.1.

Question

If Roberta computes $\left(\left(\frac{10}{3}\right) \times \left(\frac{10}{3}\right)\right) \times 9$, what is the error?

(A) 0.0

(B) 1.3

(C) 2.8

(D) 3.3

Secret Recipe

Story

Beavers are preparing for a Food Festival. They would like to bake a cake but their baker is on vacation. Keith decides to try to bake the cake. He remembers that it is important to add five essential ingredients in the correct order.

When he gets to the garden shown below, he finds a white piece of paper beside all but one ingredient. The paper shows which ingredient must be added next.



So, for example, a yellow five-petal flower must be added immediately after a pine cone. And, since there is no paper beside the strawberry, it must be added last.



Spherical Robot

Story

The BeaverBall is a toy operated by remote control which can be used to move the toy in four possible directions:



The BeaverBall operates inside a tower with its initial position shown below.



If the BeaverBall moves to a white square, it drops down one level falling directly onto the square below. The BeaverBall ignores commands that cause it to move outside the tower.

Question

Which of the following lists of directions will cause the BeaverBall to reach the GOAL?

- (A) E, W, N, W, W
- (B) E, W, N, E, S, W
- (C) E, W, E, N, S, W
- (D) E, N, W, S, N, E, W

Secret Message

Story

Agents Boris and Bertha communicate using secret messages. For example, Boris wants to communicate the following message to Bertha.

MEETBILLYBEAVERAT6

He writes each character in a 4 column grid from left-to-right and row-by-row starting from the top. He puts an X in any unused spaces in the bottom row. The result is shown below.



Then he creates the encrypted message by reading the characters from top-to-bottom and column-bycolumn starting from the left:

MBYVTEIBE6ELERXTLAAX

Bertha then uses the exact same method to reply to Boris. The encrypted message she sends to him is:

OIERKLTEILH!WBEX

Question

What was the original message sent by Bertha? (It does not contain an X.)

- (A) OKWHERETOMEET!
- (B) OKIWILLBETHERE!
- (C) WILLYOUBETHERETOO?
- (D) OKIWILLMEETHIM!

Part B

Late for Dinner

Story

Beaver Bob's friends have invited him to dinner in Beaver Town at 7:00 pm. He leaves his house (Beaver Bob's house) at 10:00 am. Bob wants to buy a gift for his friends at the Beaver Market. He also needs to pick up his son Rob at Beaver School before he drives to Beaver Town. It does not matter whether he buys the gift or picks up his son first. Bob's car currently has enough fuel left to drive for 4 hours so he also needs to visit the Gas Station to refuel his car. Once he refuels his car, he can drive for another 9 hours. The time in hours it takes to travel between locations is indicated on the map. Bob can arrive early at his friends' place but he cannot be late.



Question

Which route should Bob follow after leaving his house?

- (A) Beaverlake \rightarrow Gas Station \rightarrow Beaver Tower \rightarrow Beaver Forest \rightarrow Beaver School \rightarrow Beaver Forest \rightarrow Beaver Market \rightarrow Beaver Town
- (B) Beaver Mountains \rightarrow Beaver Tower \rightarrow Beaver Market \rightarrow Beaver Forest \rightarrow Beaver School \rightarrow Beaver Town
- (C) Beaverlake \rightarrow Gas Station \rightarrow Beaver Tower \rightarrow Beaver Market \rightarrow Beaver Forest \rightarrow Beaver School \rightarrow Beaver Town
- (D) Beaver Mountains \rightarrow Gas Station \rightarrow Beaver Tower \rightarrow Beaver Forest \rightarrow Beaver School \rightarrow Beaver Town

Log Mover

Story

Leslie the Beaver must drag logs one at a time through the system of canals and stations shown below. The logs must be dragged through canals in the direction of the arrows. The number in each arrow is the maximum total number of logs that can ever be dragged through the corresponding canal.



Question

What is the maximum number of logs that Leslie can drag from the start station to the terminal station?

- (A) 4
- (B) 5
- (C) 6
- (D) 7

Chain

Story

Cindy wrote a computer program which can be used to paint a chain of squares and triangles. The following instructions make the program draw single shapes:

- **S** draws a big square
- **s** draws a small square
- **T** draws a big triangle
- $\bullet~{\bf t}$ draws a small triangle

There is also a repeat instruction N[I] where N is a number and I is a sequence of instructions. This command makes the program repeat the instruction sequence I exactly N times in a row. For example, the instruction sequence $\mathbf{s} \ \mathbf{2}[\mathbf{T} \ \mathbf{t}] \ \mathbf{S}$ makes the program paint this chain:



Question Which instruction sequence will make the program paint the following chain? (A) s 3[t s T] t s (B) S 3[t s T] t S (C) S 3[t s T] t S (D) S 2[t s T] t S

Synchronized Robots

Story

Three robots work as a team in the warehouse shown below. The warehouse is a 6-by-6 grid.



When the team gets a direction symbol (N, S, E, or W), each robot moves one grid square in that direction at the same time.

After following a list of direction symbols, each robot picks up whatever object there might be in the same square as the robot. The robots do not pick up any other objects they encounter during their movements. The robots ignore movements that make them move outside the boundaries of the grid. For example, if we give the list N, N, S, S, E to the team, then robot A will pick up a cone, robot B will pick up a ring, and robot C will pick up a cone.

Question

What list can be sent to the team so that the team picks up exactly a sphere, a ring, and a cone?

- (A) N, E, E, E
- (B) N, E, E, S, E
- (C) N, N, S, E, N
- (D) N, E, E, S, W

Firefighter

Story

The mayor of Beaverville is looking for volunteer firefighters. A map showing the possible volunteers' homes and how they are connected by roads is shown below. He wants to ensure that every home in the town is either the home of a volunteer or is connected by a single road to the home of a volunteer.



Question

What is the minimum number of volunteers the mayor needs?

(A) 1

(B) 2

- (C) 3
- (D) 4

Part C

Find the Thief

Story

The famous Blue Diamond was stolen from a museum. A thief swapped it for a cheap green imitation diamond.



The day it was stolen, 2000 people entered the diamond room one by one. Inspector Bebro must find the thief by interviewing some of these people. He has a list of all 2000 people in the order they entered the room. The only question he can ask a person is:

Was the diamond green or blue when you saw it?

The thief will lie and say green but everyone else will tell the truth.

Question

Inspector Bebro is very clever and will use a strategy where the number of people interviewed is as small as possible. Which of the following statements can he say truthfully?

- (A) I can guarantee that I will find the thief by interviewing at most 20 people.
- (B) Interviewing 20 people will not be enough (unless I am lucky) but I can certainly find the thief by interviewing fewer than 200.
- (C) This is going to be a difficult job: I will need to interview at least 200 people, but possibly as many as 1999 people.
- (D) I cannot promise anything. If I am very unlucky, I might need to interview every single visitor.



Five types of bananas, P, Q, R, S, T, are placed in the trees with a different type in each tree. The monkey is hungry and takes the same amount of time to eat any banana. The monkey starts on a tree and begins by eating a banana. The monkey then swings to other trees and eats bananas on those trees. It takes the monkey

- three seconds to swing from the maple tree to any other tree or vice versa,
- two seconds to swing from an oak tree to a palm tree or vice versa, and
- seven seconds to swing between two oak trees or two palm trees while avoiding the maple tree along the way.

The monkey eats bananas of type P, Q, S, R, T, R, P in that order.

Question

What types of bananas can possibly be in the maple tree if the total amount of time the monkey swings is as small as possible?

- (A) P or Q or T
- (B) P or S or T
- (C) Q or S or T
- (D) Q or R or S

Room Assignment



The beavers want to choose spaces for their bedrooms. If a space is remodeled into a bedroom, all of the spaces in the lower levels connected to that bedroom can no longer be used. Each beaver has their own daily routine and goes out of their bedroom several times a day.

- Daddy beaver goes out 5 times a day.
- Mommy goes out 6 times a day.
- Brian and Liam both go out 2 times a day.
- Sarah goes out 3 times a day.

Question

If we want to minimize the total distance that all beavers travel in one day, at which level should Sarah's bedroom be located?

- (A) The 1^{st} level
- (B) The 2^{nd} level
- (C) The 3^{rd} level
- (D) The 4^{th} level

L-Game

Story

Kiki and Wiwi are playing L-Game on a 4x4 board. The player who can no longer play a piece loses. They take turns placing L-shaped pieces one at a time with Kiki playing first so that

- every piece placed by Kiki is oriented as shown below,
- every piece placed by Wiwi is oriented as shown below,
- every piece is placed entirely on the board, and
- no two pieces overlap.

The diagram below illustrates a possible board after each player has placed a piece once.



Question

Starting from an entirely empty board, how many of Kiki's nine possible first moves guarantee that Kiki will win no matter what?

- (A) 0
- (B) 1
- (C) 2
- (D) 3

Magic Potions

Story

Betaro Beaver discovered five types of magic potions with the following effects:

- makes ears longer
- makes teeth longer
- makes whiskers curly
- colours a nose white
- colours eyes white

Betaro put each magic potion into a different cup and additionally put water into a sixth cup. Betaro labelled the cups A to F and forgot to record which cup contains which magic potion!



Betaro called Taki for help. She solved the problem by experimenting on three of their other friends:

- Using the contents of cups A, B and C together produced the effects shown in Figure 1.
- Using the contents of cups A, D and E together produced the effects shown in Figure 2.
- Using the contents of cups C, D and F together produced the effects shown in Figure 3.



