Canadian Mathematics Competition

An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

2006 Fermat Contest

(Grade 11)

Wednesday, February 22, 2006

Solutions

1. Calculating, $\dfrac{1}{4 \times 5} = \dfrac{1}{20} = 0.05$.

   Answer: (B)

2. Since $2x + 3x + 4x = 12 + 9 + 6$, then $9x = 27$ or $x = 3$.

   Answer: (B)

3. Calculating each of the numerator and denominator first, $\dfrac{4^3}{10^2 - 6^2} = \dfrac{64}{100 - 36} = \dfrac{64}{64} = 1$.

   Answer: (A)

4. Calculating from the inside out, $\left(\sqrt{\sqrt{9} + \sqrt{1}}\right)^4 = \left(\sqrt{3+1}\right)^4 = \left(\sqrt{4}\right)^4 = 2^4 = 16$.

   Answer: (C)

5. Since the edge lengths of three cubes are 4, 5 and 6, then their volumes are $4^3 = 64$, $5^3 = 125$, and $6^3 = 216$, respectively.
   Thus, their mean volume is $\dfrac{64 + 125 + 216}{3} = \dfrac{405}{3} = 135$.

   Answer: (E)

6. The discount on the T-shirt is 30% of \$25, or $0.3 \times \$25 = \$7.50$.
   The discount on the jeans is 10% of \$75, or $0.1 \times \$75 = \$7.50$.
   So the total discount is $\$7.50 + \$7.50 = \$15$.

   Answer: (A)

7. If $\sqrt{2^3 \times 5 \times p}$ is an integer, then $2^3 \times 5 \times p$ is a perfect square.
   For $2^3 \times 5 \times p$ to be a perfect square, each prime factor must occur an even number of times.
   For p to be as small as possible, p must have at least one factor of 2 and at least one factor of 5.
   Thus, the smallest possible value of p is 10.
   (Checking, $2^3 \times 5 \times 10 = 400$, which is a perfect square.)

   Answer: (C)

8. From the given information, $P + Q = 16$ and $P - Q = 4$.
   Adding these two equations, we obtain $P + Q + P - Q = 16 + 4$ or $2P = 20$ or $P = 10$.

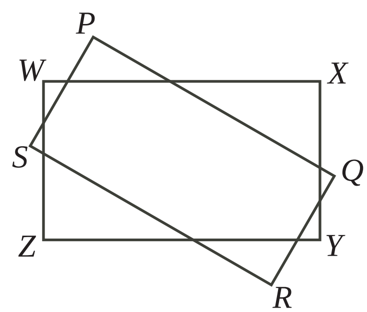
   Answer: (D)

9. *Solution 1*
   Since $\angle FAB$ is an external angle of $\triangle ABC$, then $\angle FAB = \angle ABC + \angle ACB$ or
   $70^\circ = (x^\circ + 20^\circ) + (20^\circ + x^\circ)$ or $70 = 2x + 40$ or $x = 15$.

   Solution 2
   Since $\angle FAB = 70^\circ$, then $\angle BAC = 110^\circ$.
   Looking at $\triangle ABC$, we have $\angle ABC + \angle BCA + \angle CAB = 180^\circ$, or
   $(x^\circ + 20^\circ) + (20^\circ + x^\circ) + 110^\circ = 180^\circ$ or $150 + 2x = 180$ or $x = 15$.

   Answer: (A)

10. Consider rectangles $WXYZ$ and $PQRS$.
Each of the four sides of $PQRS$ can intersect at most 2 of the sides of $WXYZ$, as any straight line can intersect at most two sides of a rectangle.
Therefore, the maximum possible number of points of intersection between the two rectangles is 8.
8 points of intersection is possible, as we can see in the diagram:



So the maximum possible number of points of intersection is 8.

Answer: (D)

11. *Solution 1*
Since $\dfrac{a}{b} = 3$, then $a = 3b$. Since $\dfrac{b}{c} = 2$, then $b = 2c$.
Since $a = 3b$ and $b = 2c$, then $a = 6c$.
Therefore, $\dfrac{a-b}{c-b} = \dfrac{6c-2c}{c-2c} = \dfrac{4c}{-c} = -4$.

Solution 2
We divide the numerator and denominator of the given expression by b to obtain $\dfrac{a-b}{c-b} = \dfrac{\frac{a}{b}-1}{\frac{c}{b}-1}$.

Since $\dfrac{b}{c} = 2$, then $\dfrac{c}{b} = \dfrac{1}{2}$.
Therefore, $\dfrac{a-b}{c-b} = \dfrac{3-1}{\frac{1}{2}-1} = \dfrac{2}{-\frac{1}{2}} = -4$.

Solution 3
Try $c = 1$. Since $\dfrac{b}{c} = 2$, then $b = 2$. Since $\dfrac{a}{b} = 3$, then $a = 6$.
Therefore, $\dfrac{a-b}{c-b} = \dfrac{6-2}{1-2} = \dfrac{4}{-1} = -4$.

Answer: (A)

12. *Solution 1*
The left side of the given equation equals $(2^4)(3^6) = 16(729) = 11\,664$.
Thus, $9(6^x) = 11\,664$ or $6^x = 1296$.
Since $6^4 = 1296$, then $x = 4$.

Solution 2
Rearranging the left side, $(2^4)(3^6) = (2^4)(3^4)(3^2) = (2 \times 3)^4(3^2) = 6^4(9) = 9(6^4)$.
Comparing with $9(6^x)$, we see that $x = 4$.

Answer: (C)

13. Let c cents be the cost of downloading 1 song in 2005.
Then the cost of downloading 1 song in 2004 was $c + 32$ cents.
The total cost in 2005 was $360c$ and the total cost in 2004 was $200(c + 32)$.
Thus, $360c = 200(c + 32)$ or $160c = 6400$ or $c = 40$ cents, and so the total cost in 2005 was $360(40) = 14400$ cents, or \$144.00.

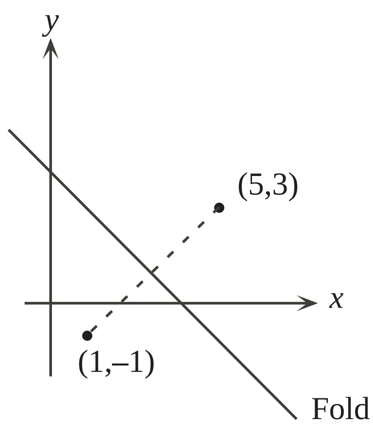
Answer: (A)

14. *Solution 1*
Adding the two given equations, we get $px + 3x = 46$ or $(p + 3)x = 46$.
Since $(x, y) = (2, -4)$ is a solution to both equations, then $x = 2$ satisfies the equation $(p + 3)x = 46$.
Thus, $2(p + 3) = 46$ or $p + 3 = 23$ or $p = 20$.

Solution 2
Substituting $(x, y) = (2, -4)$ into the second equation, we obtain $3(2) - q(-4) = 38$ or $6 + 4q = 38$ or $q = 8$.
The first equation thus becomes $px + 8y = 8$.
Substituting $(x, y) = (2, -4)$ into this new first equation, we obtain $p(2) + 8(-4) = 8$ or $2p - 32 = 8$ or $p = 20$.

Answer: (B)

15. Since the point $(5, 3)$ lands on the point $(1, -1)$ when folded, then the fold line must pass through the midpoint of these two points, namely $\left(\frac{1}{2}(5 + 1), \frac{1}{2}(3 + (-1))\right) = (3, 1)$.



Of the given possibilities, $(3, 1)$ lies only on the line $y = -x + 4$, so (D) is the answer.
(In fact, the fold line must be the perpendicular bisector of the line segment through $(5, 3)$ and $(1, -1)$. The slope of the line segment through $(5, 3)$ and $(1, -1)$ is $\dfrac{3 - (-1)}{5 - 1} = 1$, so the perpendicular bisector has slope -1.
Since the perpendicular bisector has slope -1 and passes through $(3, 1)$, then it has equation $y = -x + 4$.)

Answer: (D)

16. Since the areas of the circle and the shaded region are equal, then each is equal to half of the area of the entire rectangle, so each is equal to $\frac{1}{2}(8 \times 9) = 36$.
If the radius of the circle is r, then $\pi r^2 = 36$, so $r^2 = \frac{36}{\pi}$ or $r = \sqrt{\frac{36}{\pi}} = \frac{\sqrt{36}}{\sqrt{\pi}} = \frac{6}{\sqrt{\pi}}$.

Answer: (C)

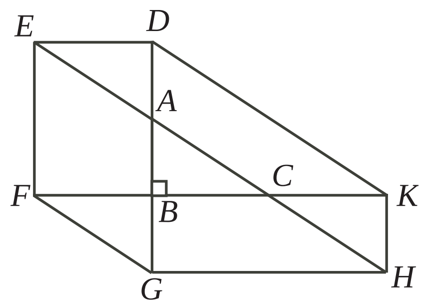
17. Since each term after the third is the sum of the preceding three terms, then, looking at the fourth term, $13 = 5 + p + q$ or $p + q = 8$.
Looking at the fifth term, $r = p + q + 13 = 8 + 13 = 21$.
Looking at the seventh term, $x = 13 + r + 40 = 13 + 21 + 40 = 74$.

Answer: (D)

18. If Georgina cycles for 6 minutes, then she cycles for $\frac{1}{10}$ of an hour.
If Georgina cycles for $\frac{1}{10}$ of an hour at 24 km/h, then she cycles a distance of 2.4 km or 2400 m.
Since the diameter of Georgina's front wheel is 0.75 m, then its circumference is $\pi d = 0.75\pi$ m.
With each rotation of the front wheel, Georgina will move 0.75π m.
So if Georgina travels 2400 m, then her front wheel rotates $\dfrac{2400}{0.75\pi} \approx 1018.59$ times, which is closest to 1020 times.

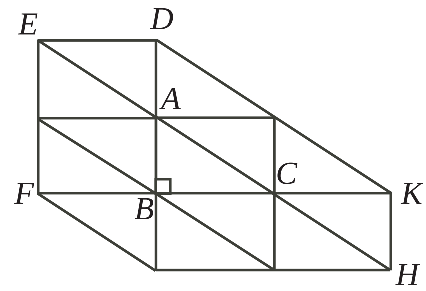
Answer: (B)

19. *Solution 1*



Let the area of $\triangle ABC$ be x.
We break up hexagon $DEFGHK$ into a number of pieces and calculate the area of each piece in terms of x.
Consider $\triangle ADE$. Since $AD = AB$, $AE = AC$ and $\angle DAE = \angle CAB$, then $\triangle ADE$ is congruent to $\triangle ABC$, so the area of $\triangle ADE$ is x.
Similarly, the area of each of $\triangle BGF$ and $\triangle CKH$ is equal to x.
Consider quadrilateral $AEFB$.
If we join this quadrilateral to $\triangle ABC$, we form $\triangle CFE$.
Since $AE = AC$, then $CE = 2CA$; similarly, $CF = 2CB$.
Since $\triangle CFE$ and $\triangle CBA$ share an angle at C and have two pairs of corresponding sides enclosing this angle in the same ratio, then $\triangle CFE$ is similar to $\triangle CBA$.
Now, the side lengths of $\triangle CFE$ are twice those of $\triangle CBA$, so the area of $\triangle CFE$ is $2^2 = 4$ times that of $\triangle CBA$, so is $4x$.
Thus, the area of quadrilateral $AEFB$ is $3x$.
Similarly, the areas of quadrilaterals $ADKC$ and $BCHG$ are $3x$.
Therefore, the area of hexagon $DEFGHK$ equals the sum of the areas of triangles ABC, ADE, BGF, and CKH, and of quadrilaterals $AEFB$, $ADKC$ and $BCHG$, so equals
$4x + 3(3x) = 13x$.
Hence, the ratio of the ratio of hexagon $DEFGHK$ to the area of $\triangle ABC$ is $13 : 1$.

Solution 2
We can triangulate hexagon $DEFGHK$ by drawing vertical line segments of length equal to that of AB, horizontal line segments of length equal to that of BC, and slanting line segments of length equal to that of AC.



Thus, we have triangulated $DEFGHK$ into 13 congruent triangles. (We can argue that each of these triangles is congruent to $\triangle ABC$ by observing that each has two perpendicular sides and noting that each has at least two sides easily seen to be equal in length to the corresponding sides in $\triangle ABC$.)
Therefore, the area of $DEFGHI$ is 13 times that of the area of $\triangle ABC$, so the ratio of the areas is $13 : 1$.

Answer: (E)

20. *Solution 1*
Suppose that Igor has removed some balls from the bag, and the remaining balls do not satisfy the required condition. What is the maximum number of balls that can remain? In order to not satisfy the required condition, either there are not 4 balls of any colour (so the maximum number is 9 balls, ie. 3 of each colour) or there are at least 4 balls of one colour, but there are not 3 of either of the other colours.
In this second case, we could have 2 balls of each of two colours, and as many as possible of the third colour. The maximum number of balls of any colour that can be in the bag is 8 (the number of yellow balls with which Igor starts). So the maximum number of balls still in the bag in this case is 12.
Therefore, if Igor removes 8 or more balls, then the remaining balls might not satisfy the required condition.
However, if Igor removes 7 or fewer balls, then the remaining balls will satisfy the required condition, since the maximum number of balls in any case which does not satisfy the condition is 12.
Therefore, the maximum possible value of N is 7.

Solution 2
Since we want to determine the maximum possible value of N, we start with the largest of the answers and rule out answers until we come to the correct answer.
If Igor removed 10 marbles, he might remove 5 red and 5 black marbles, leaving 8 yellow, 2 red, and 0 black marbles, which does not meet the required condition.
Thus, 10 is not the answer.
If Igor removed 9 marbles, he might remove 5 red and 4 black marbles, leaving 8 yellow marbles, 2 red marbles, and 1 black marble, which does not meet the required condition.
Thus, 9 is not the answer.
If Igor removed 8 marbles, he might remove 5 red and 3 black marbles, leaving 8 yellow, 2 red, and 2 black marbles, which does not meet the required condition.
Thus, 8 is not the answer.

Is 7 the answer?
There are $8 + 7 + 5 = 20$ marbles to begin with. If 7 are removed, there are 13 marbles left.

Since there are 13 marbles left, then it is not possible to have 4 or fewer marbles of each of the three colours (otherwise there would be at most 12 marbles). Thus, there are at least 5 marbles of one colour.
Could there be 2 or fewer marbles of each of the other two colours? If so, then since there are 13 marbles in total, there must be at least 9 marbles of the first colour. But there cannot be 9 or more marbles of any colour, as there were at most 8 of each colour to begin with. Therefore, there must be at least 3 of one of the other two colours of marbles.
This tells us that if 7 marbles are removed, there are at least 5 marbles of one colour and 3 of another colour, so choosing $N = 7$ marbles guarantees us the required condition.
Therefore, 7 is the maximum possible value of N.

ANSWER: (B)

21. If n is an odd integer, then each of $n-1$ and $n+1$ is even.
In fact, $n-1$ and $n+1$ are consecutive even integers, so one is a multiple of 4 and the other is divisible is 2 (since it is even).
Thus, $(n-1)(n+1)$ contains at least 3 factors of 2, which tells us that $(n-1)(n)(n+1)$ does as well, ie. is divisible by 8.
So if n is an odd integer, then $\dfrac{(n-1)(n)(n+1)}{8}$ is an integer.
(There are 39 odd integers between 2 and 80, inclusive.)
If n is an even integer, then each of $n-1$ and $n+1$ is odd.
Thus, $(n-1)(n)(n+1)$ is divisible by 8 only when n is divisible by 8.
(There are 10 multiples of 8 between 2 and 80, inclusive.)
Therefore, there are $39 + 10 = 49$ integers n, with $2 \leq n \leq 80$, such that $\dfrac{(n-1)(n)(n+1)}{8}$ is an integer.

ANSWER: (E)

22. First, we do some experimentation.
Since Celine moves small boxes faster and Quincy moves large boxes faster, then suppose Celine moves all 16 small boxes (taking 32 minutes) and Quincy moves all 10 large boxes (taking 50 minutes). Thus, they would finish in 50 minutes.
We could transfer two large boxes from Quincy to Celine, who now moves 16 small and 2 large boxes, taking 44 minutes. Quincy would then move 8 large boxes, taking 40 minutes. So they would finish in 44 minutes. (So (E) is not the answer.)
If we transfer one small box from Celine to Quincy, then Quincy moves 8 large boxes and 1 small box, taking 43 minutes, and Celine moves 15 small and 2 large boxes, taking 42 minutes. So they would finish in 43 minutes. (So (D) is not the answer.)

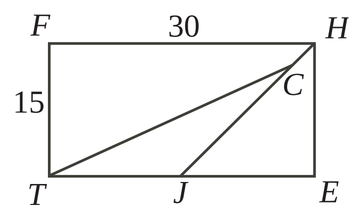
Why is 43 minutes the smallest possible total time?
Suppose that it took them at most 42 minutes to finish the job. Then the total amount of working time would be at most 84 minutes.
Suppose that Celine moves x small boxes and y large boxes, which would take $2x + 6y$ minutes. Then Quincy moves $16 - x$ small boxes and $10 - y$ large boxes, which would take $3(16 - x) + 5(10 - y) = 98 - 3x - 5y$ minutes.
Since the total working time is at most 84 minutes, then $(2x + 6y) + (98 - 3x - 5y) \leq 84$ or $14 \leq x - y$.
Since $0 \leq x \leq 16$ and $0 \leq y \leq 10$, then the possible pairs of x and y are $(16, 0)$, $(16, 1)$, $(16, 2)$, $(15, 0)$, $(15, 1)$, $(14, 0)$, which produce working times as follows:

x	y	Celine Small	Celine Large	Celine Time	Quincy Small	Quincy Large	Quincy Time
16	0	16	0	32	0	10	50
16	1	16	1	38	0	9	45
16	2	16	2	44	0	8	40
15	0	15	0	30	1	10	53
15	1	15	1	36	1	9	48
14	0	14	0	28	2	10	56

In each of these cases, while the total working time is no more than 84 minutes, it takes longer than 43 minutes to finish.
Therefore, it is impossible for them to finish in 42 minutes or less, so the earliest possible finishing time is 9:43 a.m.

ANSWER: (C)

23. *Solution 1*
Let t be the amount of time in seconds that it takes Tom to catch Jerry.



Then $TC = 5t$, since Tom runs at 5 m/s for t seconds.
Also, $JC = 3t$, since Jerry runs at 3 m/s.
We know as well that J is the midpoint of TE, so $TJ = 15 = JE$.
Since $HE = 15$, then $\angle HJE = 45°$, so $\angle TJC = 135°$.
By the cosine law,

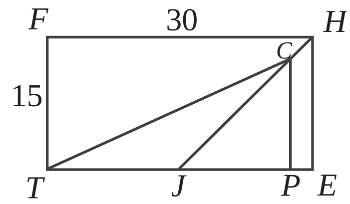
$$\begin{aligned} TC^2 &= TJ^2 + JC^2 - 2(TJ)(JC)\cos(\angle TJC) \\ (5t)^2 &= 15^2 + (3t)^2 - 2(15)(3t)\cos(135°) \\ 25t^2 &= 225 + 9t^2 - 90t\left(-\frac{1}{\sqrt{2}}\right) \\ 16t^2 - 45\sqrt{2}t - 225 &= 0 \end{aligned}$$

Using the quadratic formula, since t must be positive, we obtain

$$t = \frac{45\sqrt{2} + \sqrt{(45\sqrt{2})^2 - 4(16)(-225)}}{2(16)} = \frac{45\sqrt{2} + \sqrt{18450}}{32} \approx 6.23 \text{ seconds}$$

so it takes Tom about 6.2 seconds to catch Jerry.

Solution 2
Let t be the amount of time in seconds that it takes Tom to catch Jerry.
Then $TC = 5t$, since Tom runs at 5 m/s for t seconds.
Also, $JC = 3t$, since Jerry runs at 3 m/s.
We know as well that J is the midpoint of TE, so $TJ = 15 = JE$.
Since $HE = 15$, then $\angle HJE = 45°$.
Drop a perpendicular from C to P on JE.



Since $CJ = 3t$, $\angle CJE = 45°$ and $\angle CPJ = 90°$, then $JP = CP = \frac{1}{\sqrt{2}}(3t)$.
By the Pythagorean Theorem in $\triangle CPT$, we obtain

$$\begin{aligned} TC^2 &= (TJ + JP)^2 + CP^2 \\ (5t)^2 &= \left(15 + \tfrac{1}{\sqrt{2}}(3t)\right)^2 + \left(\tfrac{1}{\sqrt{2}}(3t)\right)^2 \\ 25t^2 &= 225 + 2(15)\left(\tfrac{1}{\sqrt{2}}(3t)\right) + \tfrac{1}{2}(9t^2) + \tfrac{1}{2}(9t^2) \\ 25t^2 &= 225 + 45\sqrt{2}t + 9t^2 \\ 16t^2 - 45\sqrt{2}t - 225 &= 0 \end{aligned}$$

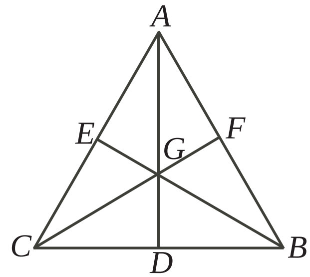
Proceeding exactly as in Solution 1, we obtain that the time is closest to 6.2 seconds.

ANSWER: (E)

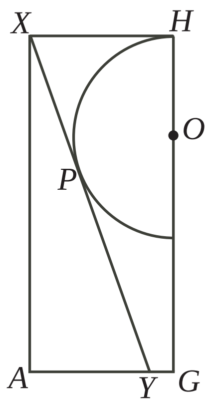
24. Since $\frac{1}{a} + \frac{1}{2a} + \frac{1}{3a} = \frac{1}{b^2 - 2b}$, then $\frac{6}{6a} + \frac{3}{6a} + \frac{2}{6a} = \frac{1}{b^2 - 2b}$ or $\frac{11}{6a} = \frac{1}{b^2 - 2b}$.
Cross-multiplying, we obtain $11(b^2 - 2b) = 6a$.
Since 11 is a divisor of the left side, then 11 must be a divisor of the right side, that is, a must be divisible by 11.
Thus, let $a = 11A$, with A a positive integer.
So we get $11(b^2 - 2b) = 6(11A)$ or $b^2 - 2b = 6A$.
Since 6 is a divisor of the right side, then 6 must be a divisor of the left side.
What is the smallest positive integer b for which 6 is a divisor of $b^2 - 2b$? We can quickly check that if b equals 1, 2, 3, 4, or 5, then $b^2 - 2b$ is not divisible by 6, but if $b = 6$, then $b^2 - 2b$ is divisible by 6.
Therefore, for the smallest values of a and b, we must have $b = 6$, so $6A = 6^2 - 2(6) = 24$, whence $A = 4$ and so $a = 11A = 44$.
Thus, the smallest possible value of $a + b$ is $44 + 6 = 50$.

ANSWER: (E)

25. First, join the centres of the bases of the three cones together.
Since the radius of the base of each cone is 50, then the distance between the centre of one base and the centre of each of the other bases is $2(50) = 100$ since the three circular bases are mutually tangent and the lines connecting their centres will pass through these points of tangency.
Thus, the triangle formed by the centres of the three bases is equilateral, with side length 100.
Now, by symmetry, the sphere will sit "in the centre" of the three cones, so the centre of the sphere will lie directly above the centroid of this triangle.
How far is the centroid from each of the vertices?
Consider equilateral triangle ABC with side length 100.
Draw in the three medians AD, BE, CF (each of which is also an altitude and an angle bisector), which intersect at G.



Since $BD = \frac{1}{2}BC = 50$ and $\angle GBD = \frac{1}{2}\angle ABC = 30^\circ$, then $\triangle BGD$ is a 30°-60°-90° triangle, so $BG = \frac{2}{\sqrt{3}}BD = \frac{100}{\sqrt{3}}$.
So the distance between each vertex of the triangle and the centroid is $\frac{100}{\sqrt{3}}$.
This tells us that the distance between the axis of each cone and the vertical line through the centre of the sphere is also $\frac{100}{\sqrt{3}}$.
Draw a vertical cross-section through the axis of one of the cones and the centre of the sphere, including half of the cone and half of the sphere.



Let A be the centre of the base of the cone (the bottom left in the diagram), G the centroid of the triangle formed by the centres, O the centre of the sphere, P the point where the sphere touches the cone, X the vertex of the cone, Y the point where the cone meets AG, and H the point where the sphere touches the plane through the vertices of the three cones.
Let r be the radius of the sphere.

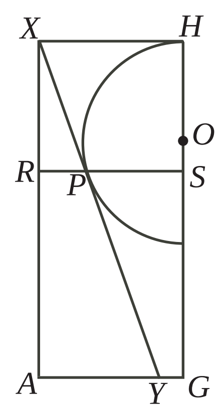
Method 1
Then we know $OH = OP = r$, $XH = AG = \frac{100}{\sqrt{3}}$ and $XA = HG = 120$.
Since the sphere is tangent to the cone at P, then OP is perpendicular to XY.
Since both XH and XP are tangent to the sphere, then $XP = XH = \frac{100}{\sqrt{3}}$.
By the Pythagorean Theorem, $XY^2 = AY^2 + AX^2 = 50^2 + 120^2 = 16900 = 130^2$, so $XY = 130$.
Thus, $PY = XY - XP = 130 - \frac{100}{\sqrt{3}}$.
Also, $OG = 120 - r$ and $GY = AG - AY = \frac{100}{\sqrt{3}} - 50$.

By the Pythagorean Theorem twice, $OP^2 + PY^2 = OY^2 = GY^2 + GO^2$, so

$$\begin{aligned} r^2 + \left(130 - \tfrac{100}{\sqrt{3}}\right)^2 &= \left(\tfrac{100}{\sqrt{3}} - 50\right)^2 + (120 - r)^2 \\ r^2 + 130^2 - \frac{2(130)(100)}{\sqrt{3}} + \frac{10000}{3} &= \frac{10000}{3} - \frac{2(50)(100)}{\sqrt{3}} + 50^2 + 120^2 - 240r + r^2 \\ 240r &= \frac{2(130-50)(100)}{\sqrt{3}} \qquad (\text{since } 130^2 = 50^2 + 120^2) \\ r &= \frac{200}{3\sqrt{3}} = \frac{200\sqrt{3}}{9} \end{aligned}$$

Evaluating, $r = \dfrac{200\sqrt{3}}{9} \approx 38.49$, so the radius is closest to 38.5.
(We could have instead calculated the radius using trigonometry, since we can calculate $\angle YXA$ using the ratio of AY to AX, and thus can calculate $\angle PXH$.
So $\tan\left(\frac{1}{2}\angle PXH\right) = \tan(\angle OXH) = \dfrac{OH}{XH} = \dfrac{r}{\frac{100}{\sqrt{3}}}$.)

<u>Method 2</u>
We know that $XH = AG = \frac{100}{\sqrt{3}}$.
Since $AX = 120$ and $AY = 50$, then the slope of XY is $\frac{-120}{50} = -\frac{12}{5}$.
Since the sphere is tangent to the cone at P, then OP is perpendicular to XY, so has slope $\frac{5}{12}$.
Draw a horizontal line through P cutting AX at R and GH at S.



Since OP has slope $\frac{5}{12}$, then we can let $OS = 5t$ and $SP = 12t$ for some real number t.
But $\triangle OSP$ is right-angled at S, so $OP = 13t$ by the Pythagorean Theorem.
Also, OH is a radius of the circle, so $OH = OP = 13t$, so $XR = HS = HO + OS = 18t$.
But the slope of XP is $-\frac{12}{5}$, so if $XR = 18t$, then $RP = \frac{5}{12}(18t) = \frac{15}{2}t$.
Therefore, $RS = RP + PS = \frac{15}{2}t + 12t = \frac{39}{2}t$ and is equal to AG which equals $\frac{100}{\sqrt{3}}$.
So $\frac{39}{2}t = \frac{100}{\sqrt{3}}$ and so the radius of the sphere, which is $13t$, equals $\dfrac{2}{3}\left(\dfrac{100}{\sqrt{3}}\right) = \dfrac{200\sqrt{3}}{9} \approx 38.49$, so the radius is closest to 38.5.

ANSWER: (D)