Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

2004 Solutions

Gauss Contest

(Grades 7 and 8)

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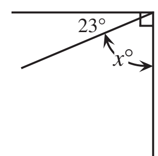
Part A

1. Simplifying,
$$\frac{10+20+30+40}{10}=\frac{100}{10}=10$$
Answer: (C)

2. Using a common denominator,
$\frac{1}{2}-\frac{1}{8}=\frac{4}{8}-\frac{1}{8}=\frac{3}{8}$
Answer: (A)

3. Seven thousand twenty-two is $7000+22=7022$.
Answer: (D)

4. From the diagram, $23^\circ + x^\circ = 90^\circ$ so $x^\circ = 67^\circ$ or $x = 67$.



Answer: (C)

5. Since Sally was 7 years old five years ago, then she is 12 years old today. Thus, in two more years, she will be 14.
Answer: (B)

6. Since Stuart earns 5 reward points for every \$25 he spends, then when he spends \$200, he earns $\frac{200}{25}\times 5 = 8\times 5 = 40$ points.
Answer: (C)

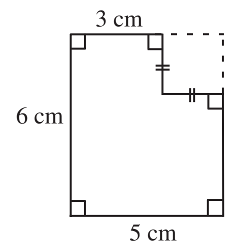
7. Using a calculator, $\frac{8}{9}=0.888...$, $\frac{7}{8}=0.875$, $\frac{66}{77}=\frac{6}{7}=0.857...$, $\frac{55}{66}=\frac{5}{6}=0.833...$, $\frac{4}{5}=0.8$, so $\frac{8}{9}=0.888...$ is the largest.
Answer: (A)

8. There are 6 balls in the box. 5 of the balls in the box are not grey. Therefore, the probability of selecting a ball that is not grey is $\frac{5}{6}$.
Answer: (E)

9. The sum of the numbers in the second column is $19+15+11=45$, so the sum of the numbers in any row column or diagonal is 45. The sum of the two numbers already in the first row is 33, so the third number in the first row (in the upper right corner) must be 12. Finally, the diagonal from bottom left to top right has x, 15 and 12, so $x+15+12=45$ or $x=18$.
Answer: (E)

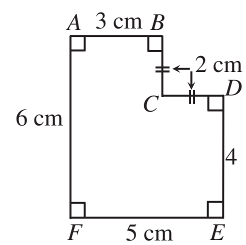
10. *Solution 1*

We notice that if we complete the given figure to form a rectangle, then the perimeter of this rectangle and the original figure are identical. Therefore, the perimeter is $2 \times 5 + 2 \times 6 = 22$ cm.



Solution 2

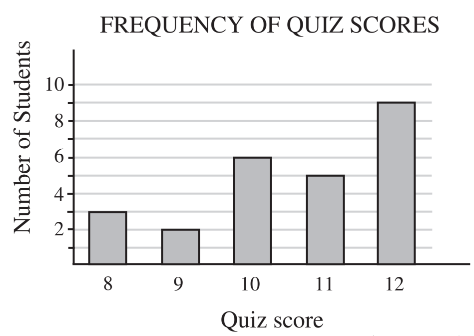
Since the width of the figure is 5 cm, then $AB + CD = 5$ cm, so $CD = BC = 2$ cm.
Since the height of the figure is 6 cm, then $BC + DE = 6$ cm, so $DE = 4$ cm.
Therefore, the perimeter is $3 + 2 + 2 + 4 + 5 + 6 = 22$ cm.



Answer: (E)

Part B

11. When we list the quiz scores in ascending order, including repetition, we get 8, 8, 8, 9, 9, 10, 10, 10, 10, 10, 10, 11, 11, 11, 11, 11, 12, 12, 12, 12, 12, 12, 12, 12, 12. Since there are 25 scores, the middle score is the 13th along, so the median is 11.



Answer: (D)

12. In travelling between the two lakes, the total change in elevation is $174.28 - 75.00 = 99.28$ m. Since this change occurs over 8 hours, the average change in elevation per hour is $\frac{99.28 \text{ m}}{8 \text{ h}} = 12.41 \text{ m / h}$.

Answer: (A)

13. We make a chart of the pairs of positive integers which sum to 11 and their corresponding products:

First integer	Second integer	Product
1	10	10
2	9	18
3	8	24
4	7	28
5	6	30

so the greatest possible product is 30.

Answer: (E)

14. Evaluating the exponents, $3^2 = 9$ and $3^3 = 27$, so the even whole numbers between the two given numbers are the even whole numbers from 10 to 26, inclusive. These are 10, 12, 14, 16, 18, 20, 22, 24, and 26, so there are 9 of them.

Answer: (A)

15. If $P = 1000$ and $Q = 0.01$, then

$$P + Q = 1000 + 0.01 = 1000.01$$
$$P \times Q = 1000 \times 0.01 = 10$$
$$\frac{P}{Q} = \frac{1000}{0.01} = 100000$$
$$\frac{Q}{P} = \frac{0.01}{1000} = 0.00001$$
$$P - Q = 1000 - 0.01 = 999.99$$

so the largest is $\frac{P}{Q}$.

Answer: (C)

16. The volume of the box of $40 \times 60 \times 80 = 192000\text{ cm}^3$. The volume of each of the blocks is $20 \times 30 \times 40 = 24000\text{ cm}^3$. Therefore, the maximum number of blocks that can fit inside the box is $\frac{192000\text{ cm}^3}{24000\text{ cm}^3} = 8$. 8 blocks can indeed be fit inside this box. Can you see how?

Answer: (D)

17 In the recipe, the ratio of volume of flour to volume of shortening is 5 : 1. Since she uses $\frac{2}{3}$ cup of shortening, then to keep the same ratio as called for in the recipe, she must use $5 \times \frac{2}{3} = \frac{10}{3} = 3\frac{1}{3}$ cups of flour.

Answer: (B)

18. The rectangular prism in the diagram is made up of 12 cubes. We are able to see 10 of these 12 cubes. One of the two missing cubes is white and the other is black. Since the four blocks of each colour are attached together to form a piece, then the middle block in the back row in the bottom layer must be white, so the missing black block is the leftmost block of the back row in the bottom layer. Thus, the leftmost block in the back row in the top layer is attached to all three of the other black blocks, so the shape of the black piece is (A). (This is the only one of the 5 possibilities where one block is attached to three other blocks.)

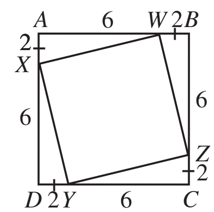
Answer: (A)

19. Since the number is divisible by $8 = 2^3$, by $12 = 2^2 \times 3$, and by $18 = 2 \times 3^2$, then the number must have at least three factors of 2 and two factors of 3, so the number must be divisible by $2^3 \times 3^2 = 72$. Since the number is a two-digit number which is divisible by 72, it must be 72 (it cannot have more than two digits), so it is between 60 and 79.

Answer: (D)

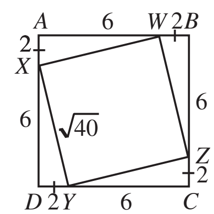
20. *Solution 1*

Since the area of square *ABCD* is 64, then the side length of square *ABCD* is 8. Since $AX = BW = CZ = DY = 2$, then $AW = BZ = CY = DX = 6$. Thus, each of triangles *XAW*, *WBZ*, *ZCY* and *YDX* is right-angled with one leg of length 2 and the other of length 6. Therefore, each of these four triangles has area $\frac{1}{2}(2)(6) = 6$. Therefore, the area of square *WXYZ* is equal to the area of square *ABCD* minus the sum of the areas of the four triangles, or $64 - 4(6) = 40$.



Solution 2

Since the area of square *ABCD* is 64, then the side length of square *ABCD* is 8. Since $AX = BW = CZ = DY = 2$, then $AW = BZ = CY = DX = 6$.
By the Pythagorean Theorem,
$XW = WZ = ZY = YX = \sqrt{2^2 + 6^2} = \sqrt{4 + 36} = \sqrt{40}$.
Therefore, the area of square *WXYZ* is $\left(\sqrt{40}\right)^2 = 40$.



Answer: (D)

Part C

21. In the diagram, we will refer to the horizontal dimension as the width of the room and the vertical dimension as the length of the room. Since the living room is square and has an area of 16 m^2, then it has a length of 4 m and a width of 4 m. Since the laundry room is square and has an area of 4 m^2, then it has a length of 2 m and a width of 2 m. Since the dining room has a length of 4 m (the same as the length of the living room) and an area of 24 m^2, then it has a width of 6 m. Thus, the entire ground floor has a width of 10 m, and so the kitchen has a width of 8 m (since the width of the laundry room is 2 m) and a length of 2 m(since the length of the laundry room is 2 m), and so the kitchen has an area of 16 m^2.

Answer: (B)

22. Let the volume of a large glass be *L* and of a small glass be *S*. Since the jug can exactly fill either 9 small glasses and 4 large glasses, or 6 small glasses and 6 large glasses, then $9S + 4L = 6S + 6L$ or $3S = 2L$. In other words, the volume of 3 small glasses equals the volume of 2 large glasses. (We can also see this without using algebra – if we compare the two cases, we can see that if we remove 3 small glasses then we increase the volume by 2 large glasses.) Therefore, the volume of 9 small glasses equals the volume of 6 large glasses. Thus, the volume of 9 small glasses and 4 large glasses equals the volume of 6 large glasses and 4 large glasses, or 10 large glasses in total, and so the jug can fill 10 large glasses in total.

Answer: (C)

23. In her 40 minutes (or $\frac{2}{3}$ of an hour) on city roads driving at an average speed of 45 km/h, Sharon drives $\left(\frac{2}{3}\text{ h}\right)\times(45\text{ km/h})=30\text{ km}$. So the distance that she drives on the highway must be $59\text{ km}-30\text{ km}=29\text{ km}$. Since she drives this distance in 20 minutes (or $\frac{1}{3}$ of an hour), then her average speed on the highway is $\dfrac{29\text{ km}}{\frac{1}{3}\text{ h}}=(29\times 3)\text{ km/h}=87\text{ km/h}$.

Answer: (C)

24. We consider each possible number of silver medals starting with 8.
Could she have won 8 silver medals? This would account for 24 points in 8 events, but since she won 27 points in 8 events, this is not possible.
Could she have won 7 silver medals? This would account for 21 points in 7 events, and so in the remaining 1 event, she would have won 6 points, which is impossible, since she could not score more than 5 points (a gold medal) on this event.
Could she have won 6 silver medals? This would account for 18 points in 6 events, and so in the remaining 2 events, she would have won 9 points, which is impossible, since we cannot combine either two 5s, two 1s or a 1 and a 5 to get 9.
Could she have won 5 silver medals? This would account for 15 points in 5 events, and so in the remaining 3 events, she would have won 12 points, which is impossible. (Try combining up to three 5s and 1s to get 12. We need at least two 5s and two 1s to make 12.)
Could she have won 4 silver medals? This would account for 12 points in 4 events, and so in the remaining 4 events, she would have won 15 points. This is possible – she could win gold on 3 of the 4 remaining events (for 15 points in total) and no medal on the last event (there are 6 competitors and only 3 medals for each event, so there are competitors who do not win medals).
Thus, the maximum number of silver medals she could have won is 4.

Answer: (D)

Answer: (D)

25. *Solution 1*
Start with a grid with two columns and ten rows. There are 10 ways to place the domino horizontally (one in each row) and 18 ways to place the column vertically (nine in each column), so 28 ways overall.
How many more positions are added when a new column is added? When a new column is added, there are 9 new vertical positions (since the column has ten squares) and 10 new horizontal positions (one per row overlapping the new column and the previous column). So there are 19 new positions added.
How many times do we have to add 19 to 28 to get to 2004? In other words, how many times does 19 divide into $2004-28=1976$? Well, $1976\div 19=104$, so we have to add 104 new columns to the original 2 columns, for 106 columns in total.

Solution 2
Let the number of columns be n.
In each column, there are 9 positions for the domino (overlapping squares 1 and 2, 2 and 3, 3 and 4, and so on, down to 9 and 10).
In each row, there are $n-1$ positions for the domino (overlapping squares 1 and 2, 2 and 3, 3 and 4, and so on, along to $n-1$ and n).
Therefore, the total number of positions for the domino equals the number of rows times the number of positions per row plus the number of columns times the number of positions per column, or $n(9)+10(n-1)=19n-10$. We want this to equal 2004, so $19n-10=2004$ or $19n=2014$ or $n=106$. Thus, there are 106 columns.

Answer: (B)

Part A

1. 25% of 2004 is $\frac{1}{4}$ of 2004, or 501.

Answer: (B)

2. Using a common denominator,
$$\tfrac{1}{2}+\tfrac{3}{4}-\tfrac{5}{8}=\tfrac{4}{8}+\tfrac{6}{8}-\tfrac{5}{8}=\tfrac{5}{8}$$

Answer: (C)

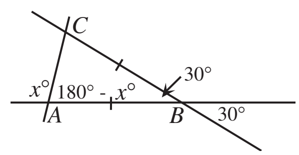
3. Rewriting the given integer,
$$800\,670=800\,000+600+70=8\times10^5+6\times10^2+7\times10^1$$
so $x=5$, $y=2$ and $z=1$, which gives $x+y+z=8$.

Answer: (B)

4. Rewriting the right side with a common denominator,
$$\frac{7863}{13}=\frac{604\times13+\square}{13}=\frac{7852+\square}{13}$$
Therefore, $\square=11$.

Answer: (A)

5. In the diagram, $\angle ABC=\angle XBY=30^\circ$ since they are opposite angles.
Also, $\angle BAC=180^\circ-x^\circ$ by supplementary angles, and so $\angle BCA=180^\circ-x^\circ$ because triangle ABC is isosceles. Looking at the sum of the angles in triangle ABC, we have $180^\circ-x^\circ+180^\circ-x^\circ+30^\circ=180^\circ$
$$210^\circ=2x^\circ$$
$$x=105$$



Answer: (D)

6. Since the perimeter of each of the small equilateral triangles is 6 cm, then the side length of each of these triangles is 2 cm. Since there are three of the small triangles along each side of triangle of *ABC*, then the side length of triangle *ABC* is 6 cm, and so its perimeter is 18 cm.

Answer: (A)

7. If $x=-4$ and $y=4$, then
$$\frac{x}{y}=\frac{-4}{4}=-1$$
$$y-1=4-1=3$$
$$x-1=-4-1=-5$$
$$-xy=-(-4)(4)=16$$
$$x+y=-4+4=0$$
Thus, $-xy$ is the largest.

Answer: (D)

8. When two coins are tossed, there are four equally likely outcomes: HEADS and HEADS, HEADS and TAILS, TAILS and HEADS, and TAILS and TAILS. One of these four outcomes has both coins landing as HEADS. Thus, the probability is $\frac{1}{4}$.

Answer: (E)

9. The water surface has an elevation of +180 m, and the lowest point of the lake floor has an elevation of –220 m. Therefore, the actual depth of the lake at this point is $180-(-220)=400\,\text{m}$.

Answer: (D)

10. We make a chart of the pairs of positive integers which sum to 11 and their corresponding products:

First integer	Second integer	Product
1	10	10
2	9	18
3	8	24
4	7	28
5	6	30

so the greatest possible product is 30.

Answer: (E)

Part B

11. To walk 1.5 km, Ruth takes $\frac{1.5\text{ km}}{5\text{ km / h}}=0.3\text{ h}=18\text{ min}$.

Answer: (C)

12. Computing each of the first and fourth of the numbers, we have the four numbers $\sqrt{36}=6$, 35.2, 35.19 and $5^2=25$. Arranging these in increasing order gives 6, 25, 35.19, 35.2, or $\sqrt{36}=6$, $5^2=25$, 35.19, 35.2.

Answer: (D)

13. We number the trees from 1 to 13, with tree number 1 being closest to Trina's house and tree number 13 being closest to her school. On the way to school, she puts a chalk mark on trees 1, 3, 5, 7, 9, 11, 13. On her way home, she puts a chalk mark on trees 13, 10, 7, 4, 1. This leaves trees 2, 6, 8, 12 without chalk marks.

Answer: (B)

14. The rectangular prism in the diagram is made up of 12 cubes. We are able to see 10 of these 12 cubes. One of the two missing cubes is white and the other is black. Since the four blocks of each colour are attached together to form a piece, then the middle block in the back row in the bottom layer must be white, so the missing black block is the leftmost block of the back row in the bottom layer. Thus, the leftmost block in the back row in the top layer is attached to all three of the other black blocks, so the shape of the black piece is (A). (This is the only one of the 5 possibilities where one block is attached to three other blocks.)

Answer: (A)

15. This solid can be pictured as a rectangular prism with dimensions 4 by 5 by 6 with a rectangular prism with dimensions 1 by 2 by 4 removed. Therefore, the volume is $4 \times 5 \times 6 - 1 \times 2 \times 4 = 120 - 8 = 112$.

Answer: (B)

16. Since the number is divisible by $8 = 2^3$, by $12 = 2^2 \times 3$, and by $18 = 2 \times 3^2$, then the number must have at least three factors of 2 and two factors of 3, so the number must be divisible by $2^3 \times 3^2 = 72$. Since the number is a two-digit number which is divisible by 72, it must be 72, so it is between 60 and 79.

Answer: (D)

17. Since $2^3 = 8$ and $2^a = 8$, then $a = 3$. Since $a = 3$ and $a = 3c$, then $c = 1$.

Answer: (C)

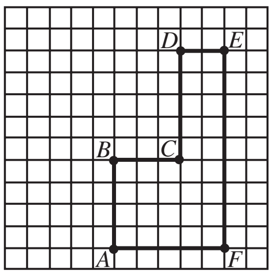
18. Since the range is unchanged after a score is removed, then the score that we remove cannot be the smallest or the largest (since each of these occurs only once). Thus, neither the 6 nor the 10 is removed.
Since the mode is unchanged after a score is removed, then the score that we remove cannot be the most frequently occurring. Thus, the score removed is not an 8.
Therefore, either a 7 or a 9 is removed.
Since we wish to increase the average, we remove the smaller of the two numbers, ie. the 7. (We could calculate that before any number is removed, the average is 7.875, if the 7 is removed, the average is 8, and if the 9 is removed, the average is 7.714.)

Answer: (B)

19. Since the numerical values of CAT and CAR are 8 and 12, then the value of R must be 4 more than the value of T.
Therefore, the value of BAR is 4 more than the value of BAT, so BAR has a numerical value of 10.

Answer: (A)

20. To get from A to E, we go right 5 and up 9, so $AE = \sqrt{5^2 + 9^2} = \sqrt{106} \approx 10.30$, by the Pythagorean Theorem.
To get from C to F, we go right 2 and down 4, so $CF = \sqrt{2^2 + 4^2} = \sqrt{20} \approx 4.47$, and so $CD + CF \approx 5 + 4.47 = 9.47$.
To get from A to C, we go right 3 and up 4, so $AC = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$, and so $AC + CF \approx 5 + 4.47 = 9.47$.
To get from F to D, we go left 2 and up 9, so $FD = \sqrt{2^2 + 9^2} = \sqrt{85} \approx 9.22$.
To get from C to E, we go right 2 and up 5, so $CE = \sqrt{2^2 + 5^2} = \sqrt{29} \approx 5.39$, and so $AC + CE \approx 5 + 5.39 = 10.39$.
Therefore, the longest of these five lengths is $AC + CE$.



Answer: (E)

Part C

21. The scale of the map is equal to the ratio of a distance on the map to the actual distance. Since the distance between Saint John and St. John's is 21 cm on the map and 1050 km in reality, then the scale of the map is equal to

$$21 \text{ cm} : 1050 \text{ km} = 0.21 \text{ m} : 1\,050\,000 \text{ m} = 21 : 105\,000\,000 = 1 : 5\,000\,000$$

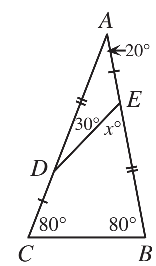
Answer: (E)

22. *Solution 1*
When the pouring stops, $\frac{1}{4}$ of the water in the bottle has been transferred to the glass. This represents $\frac{3}{4}$ of the volume of the glass. Therefore, the volume of the bottle is three times the volume of the glass, so the volume of the glass is 0.5 L.

Solution 2
When the pouring stops, $\frac{1}{4}$ of the water in the bottle or $\frac{1}{4}\times 1.5 = 0.375$ L of water is in the glass. Since this represents $\frac{3}{4}$ of the volume of the glass, then the volume of the glass is $\frac{4}{3}\times 0.375 = 0.5$ L.

Answer: (A)

23. From the diagram, $BE = AD$ and $AE = CD$, so
$AC = AD + CD = BE + AE = AB$ so triangle ABC is isosceles.
Therefore, $\angle ACB = \angle ABC = 80^\circ$ and so
$\angle BAC = 180^\circ - \angle ABC - \angle ACB = 180^\circ - 80^\circ - 80^\circ = 20^\circ$.
Considering triangle AED next,
$\angle AED = 180^\circ - \angle ADE - \angle EAD = 180^\circ - 30^\circ - 20^\circ = 130^\circ$.
But $x^\circ = 180^\circ - \angle AED = 180^\circ - 130^\circ = 50^\circ$ so $x = 50$.



Answer: (C)

24. Since x has digits ABC, then $x = 100A + 10B + C$.
Since y has digits CBA, then $y = 100C + 10B + A$.
Since $x - y = 495$, then

$$\begin{aligned}(100A + 10B + C) - (100C + 10B + A) &= 495\\ 99A - 99C &= 495\\ 99(A - C) &= 495\\ A - C &= 5\end{aligned}$$

and there is no restriction on B.

Thus, there are 10 possibilities for B (0 through 9) and for each of these possibilities we could have A and C equal to 6 and 1, 7 and 2, 8 and 3, or 9 and 4. (For example, $873 - 378 = 495$.)
Therefore, there are 40 possibilities for x.

Answer: (B)

25. Consider the block as n layers each having 11 rows and 10 columns.
First, we consider positions of the 2 by 1 by 1 block which are entirely contained in one layer. In each layer, there are 9 possible positions for the 2 by 1 by 1 block in each row (crossing columns 1 and 2, 2 and 3, 3 and 4, and so on, up to 9 and 10), and there are 10 possible positions in each column (crossing rows 1 and 2, 2 and 3, 3 and 4, and so on, up to 10 and 11). Therefore, within each layer, there are $11(9)+10(10)=199$ positions for the 2 by 1 by 1 block. In the large block, there are thus $199n$ positions of this type for the 2 by 1 by 1 block, since there are n layers.

Next, we consider positions of the 2 by 1 by 1 block which cross between two layers. Since each layer has 110 blocks (in 11 rows and 10 columns) then there are 110 positions for the 2 by 1 by 1 block between each pair of touching layers. Since there are $n-1$ pairs of touching layers, then there are $110(n-1)$ positions of this type.

Thus, overall we have 2362 total positions, so

$$\begin{aligned} 199n+110(n-1) &= 2362 \\ 309n-110 &= 2362 \\ 309n &= 2472 \\ n &= 8 \end{aligned}$$

Answer: (B)