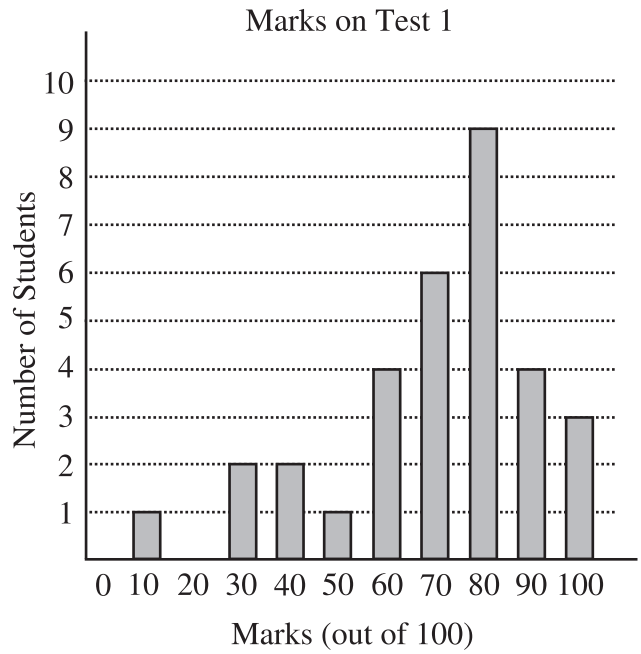
2003 Fryer Contest *(Grade 9)*

Wednesday, April 16, 2003

1. (a) The marks of 32 mathematics students on Test 1 are all multiples of 10 and are shown on the bar graph. What was the average (mean) mark of the 32 students in the class?



   (b) After his first 6 tests, Paul has an average of 86. What will his average be if he scores 100 on his next test?

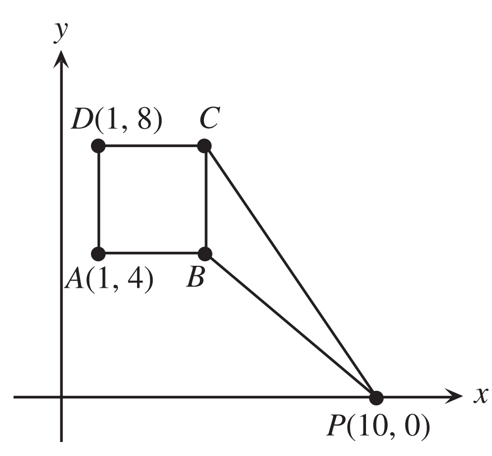
   (c) Later in the year, Mary realizes that she needs a mark of 100 on the next test in order to achieve an average of 90 for all her tests. However, if she gets a mark of 70 on the next test, her average will be 87. After she writes the next test, how many tests will she have written?

2. In a game, Xavier and Yolanda take turns calling out whole numbers. The first number called must be a whole number between and including 1 and 9. Each number called after the first must be a whole number which is 1 to 10 greater than the previous number called.

   (a) The first time the game is played, the person who calls the number 15 is the winner. Explain why Xavier has a winning strategy if he goes first and calls 4.

   (b) The second time the game is played, the person who calls the number 50 is the winner. If Xavier goes first, how does he guarantee that he will win?

3. In the diagram, *ABCD* is a square and the coordinates of *A* and *D* are as shown.



   (a) The point *P* has coordinates $(10,0)$. Show that the area of triangle *PCB* is 10.

   (b) Point $E(a,0)$ is on the *x*-axis such that triangle *CBE* lies entirely outside square *ABCD*. If the area of the triangle is equal to the area of the square, what is the value of *a*?

   (c) Show that there is no point *F* on the *x*-axis for which the area of triangle *ABF* is equal to the area of square *ABCD*.

4. For the set of numbers $\{1,10,100\}$ we can obtain 7 *distinct* numbers as totals of one or more elements of the set. These totals are 1, 10, 100, $1+10=11$, $1+100=101$, $10+100=110$, and $1+10+100=111$. The "power-sum" of this set is the *sum* of these totals, in this case, 444.

   (a) How many distinct numbers may be obtained as a sum of one or more different numbers from the set $\{1,10,100,1000\}$? Calculate the power-sum for this set.

   (b) Determine the power-sum of the set $\{1,10,100,1000,10\,000,100\,000,1\,000\,000\}$.

over ...

Extensions (Attempt these only when you have completed as much as possible of the four main problems.)

Extension to Problem 1:
Mary's teacher records the final marks of the 32 students. The teacher calculates that, for the entire class, the median mark is 80. The teacher also calculates that the difference between the highest and lowest marks is 40 and calculates that the average mark for the entire class is 58. Show that the teacher has made a calculation error.

Extension to Problem 2:
In the game described in (b), the target number was 50. For what different values of the target number is it guaranteed that Yolanda will have a winning strategy if Xavier goes first?

Extension to Problem 3:
G is a point on the line passing through the points $M(0, 8)$ and $N(3, 10)$ such that ΔDCG lies entirely outside the square. If the area of ΔDCG is equal to the area of the square, determine the coordinates of G.

Extension to Problem 4:
Consider the set $\{1, 2, 3, 6, 12, 24, 48, 96\}$. How many different totals are now possible if a total is defined as the sum of 1 or more elements of a set?