Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

2002 Solutions

Gauss Contest

(Grades 7 and 8)

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2002 Gauss Solutions

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Part A

1. When the numbers 8, 3, 5, 0, 1 are arranged from smallest to largest, the middle number is

   (**A**) 5 (**B**) 8 (**C**) 3 (**D**) 0 (**E**) 1

   Solution

   If we rearrange the given numbers from smallest to largest, we would have 0, 1, 3, 5, 8.
   The middle number is 3. ANSWER: (C)

2. The value of $0.9 + 0.99$ is

   (**A**) 0.999 (**B**) 1.89 (**C**) 1.08 (**D**) 1.98 (**E**) 0.89

   Solution

   Adding,

$$\begin{array}{r} 0.9 \\ +\ 0.99 \\ \hline 1.89 \end{array}$$

   ANSWER: (B)

3. $\dfrac{2+1}{7+6}$ equals

   (**A**) $\frac{3}{13}$ (**B**) $\frac{21}{76}$ (**C**) $\frac{1}{21}$ (**D**) $\frac{2}{13}$ (**E**) $\frac{1}{14}$

   Solution

   Evaluating,

$$\frac{2+1}{7+6} = \frac{3}{13}.$$

   ANSWER: (A)

4. 20% of 20 is equal to

   (**A**) 400 (**B**) 100 (**C**) 5 (**D**) 2 (**E**) 4

   Solution

   20% of 20 equals $0.2 \times 20 = 4$. Alternatively, 20% of 20 is $\frac{1}{5}$ of 20, or 4.

   ANSWER: (E)

5. Tyesha earns \$5 per hour babysitting, and babysits for 7 hours in a particular week. If she starts the week with \$20 in her bank account, deposits all she earns into her account, and does not withdraw any money, the amount she has in her account at the end of the week is

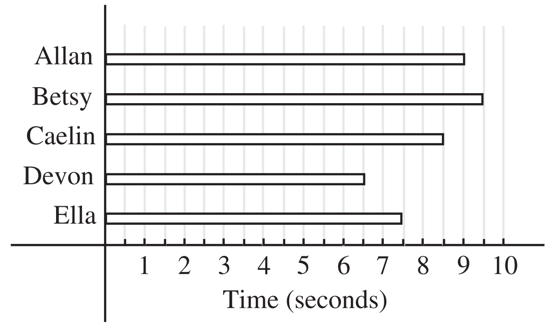
   (**A**) \$35 (**B**) \$20 (**C**) \$45 (**D**) \$55 (**E**) \$65

Solution

If Tyesha earns \$5 per hour and works for 7 hours, then she earns $7 \times \$5 = \$35$ in total. If she started with \$20 in her bank account and adds the \$35, she will have $\$20 + \$35 = \$55$ in her account. ANSWER: (D)

6. Five rats competed in a 25 metre race. The graph shows the time that each rat took to complete the race. Which rat won the race?

(**A**) Allan (**B**) Betsy (**C**) Caelin
(**D**) Devon (**E**) Ella



Solution

Since each of the rats completed the race, then the rat taking the least amount of time won the race. Since Devon took the least amount of time, she was the winner. ANSWER: (D)

7. The mean (average) of the numbers 12, 14, 16, and 18, is

(**A**) 30 (**B**) 60 (**C**) 17 (**D**) 13 (**E**) 15

Solution

The mean of the given numbers is

$$\frac{12+14+16+18}{4} = \frac{60}{4} = 15.$$

ANSWER: (E)

8. If $P = 1$ and $Q = 2$, which of the following expressions is **not** equal to an integer?

(**A**) $P+Q$ (**B**) $P \times Q$ (**C**) $\frac{P}{Q}$ (**D**) $\frac{Q}{P}$ (**E**) P^Q

Solution

Evaluating the choices,

(A) $P+Q=3$ **(B)** $P \times Q = 2$ **(C)** $\frac{P}{Q} = \frac{1}{2}$ **(D)** $\frac{Q}{P} = \frac{2}{1} = 2$ **(E)** $P^Q = 1^2 = 1$

ANSWER: (C)

9. Four friends equally shared $\frac{3}{4}$ of a pizza, which was left over after a party. What fraction of a whole pizza did each friend get?

(**A**) $\frac{3}{8}$ (**B**) $\frac{3}{16}$ (**C**) $\frac{1}{12}$ (**D**) $\frac{1}{16}$ (**E**) $\frac{1}{8}$

Solution

If $\frac{3}{4}$ of a pizza was shared by 4 friends, they would each receive $\frac{1}{4}$ of $\frac{3}{4}$, or $\frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$ of the pizza.

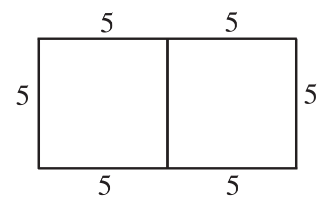
ANSWER: (B)

10. Two squares, each with an area of $25\ \text{cm}^2$, are placed side by side to form a rectangle. What is the perimeter of this rectangle?

(**A**) 30 cm (**B**) 25 cm (**C**) 50 cm (**D**) 20 cm (**E**) 15 cm

Solution

If the two squares are placed side by side, the rectangle shown would be formed.
The perimeter of this newly formed rectangle is 30 cm.



ANSWER: (A)

Part B

11. After running 25% of a race, Giselle had run 50 metres. How long was the race, in metres?

(**A**) 100 (**B**) 1250 (**C**) 200 (**D**) 12.5 (**E**) 400

Solution

If 25% of a race is 50 metres, then 100% of the race is $\frac{100}{25} \times 50 = 200$ metres.

ANSWER: (C)

12. Qaddama is 6 years older than Jack. Jack is 3 years younger than Doug. If Qaddama is 19 years old, how old is Doug?

(**A**) 17 (**B**) 16 (**C**) 10 (**D**) 18 (**E**) 15

Solution

If Qaddama is 6 years older than Jack and she is 19 years old, then Jack is 13 years old. If Jack is 3 years younger than Doug, then Doug must be 16 years of age. ANSWER: (B)

13. A palindrome is a positive integer whose digits are the same when read forwards or backwards. For example, 2002 is a palindrome. What is the smallest number which can be added to 2002 to produce a larger palindrome?

(**A**) 11 (**B**) 110 (**C**) 108 (**D**) 18 (**E**) 1001

Solution

The best way to analyze this problem is by asking the question, "What is the next palindrome bigger than 2002?" Since the required palindrome should be of the form $2aa2$, where the middle two digits (both a) do not equal 0, it must be the number 2112. Thus, the number that must be added to 2002 is $2112 - 2002 = 110$. Answer: (B)

14. The first six letters of the alphabet are assigned values A = 1, B = 2, C = 3, D = 4, E = 5, and F = 6. The value of a word equals the sum of the values of its letters. For example, the value of BEEF is 2 + 5 + 5 + 6 = 18. Which of the following words has the greatest value?
(**A**) BEEF (**B**) FADE (**C**) FEED (**D**) FACE (**E**) DEAF

Solution

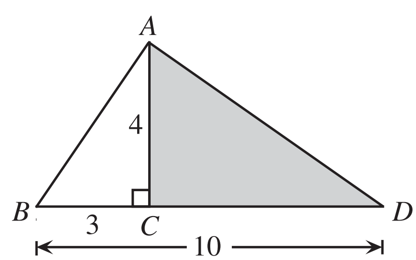
Each of the five given words contains both an "E" and an "F", so we can eliminate these letters for the purposes of making the comparison. So after eliminating these letters we are looking at the 5 "words",

(A) BE **(B)** AD **(C)** ED **(D)** AC **(E)** DA

The highest value of these five words is ED, which has a value of 9, thus implying that FEED has the highest value of the original five words.

Alternatively, we could have calculated the value of each of the five words, and again seen that FEED has the highest value. Answer: (C)

15. In the diagram, $AC = 4$, $BC = 3$, and $BD = 10$. The area of the shaded triangle is
(**A**) 14 (**B**) 20 (**C**) 28
(**D**) 25 (**E**) 12



Solutionn

If $BD = 10$ and $BC = 3$, then $CD = 7$. The area of the shaded triangle is $\frac{1}{2}(7)(4) = 14$.

Answer: (A)

16. In the following equations, the letters a, b and c represent different numbers.

$$1^3 = 1$$
$$a^3 = 1 + 7$$
$$3^3 = 1 + 7 + b$$
$$4^3 = 1 + 7 + c$$

The numerical value of $a + b + c$ is
(**A**) 58 (**B**) 110 (**C**) 75 (**D**) 77 (**E**) 79

Solution

Since $2^3 = 8 = 1 + 7$, then $a = 2$.

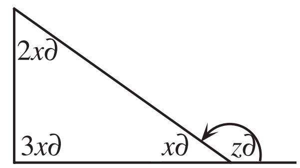
Since $3^3 = 27$, then $27 = 8 + b$ or $b = 19$.

Since $4^3 = 64$, then $64 = 8 + c$ or $c = 56$.

Thus, $a + b + c = 2 + 19 + 56 = 77$. ANSWER: (D)

17. In the diagram, the value of z is

(**A**) 150 (**B**) 180 (**C**) 60
(**D**) 90 (**E**) 120



Solution

Since the angles of a triangle add to 180∂,

$$2x\partial + 3x\partial + x\partial = 180\partial$$
$$6x\partial = 180\partial$$
$$x = 30$$

Now since the angles $x\partial$ and $z\partial$ together form a straight line, the

$$x\partial + z\partial = 180\partial$$
$$30\partial + z\partial = 180\partial$$
$$z = 150$$

ANSWER: (A)

18. A perfect number is an integer that is equal to the sum of all of its positive divisors, except itself. For example, 28 is a perfect number because $28 = 1 + 2 + 4 + 7 + 14$. Which of the following is a perfect number?

(**A**) 10 (**B**) 13 (**C**) 6 (**D**) 8 (**E**) 9

Solution

We must check each of the answers:

	Number	Positive divisors	Sum of all positive divisors except itself
(A)	10	1, 2, 5, 10	$1 + 2 + 5 = 8$
(B)	13	1, 13	1
(C)	6	1, 2, 3, 6	$1 + 2 + 3 = 6$
(D)	8	1, 2, 4, 8	$1 + 2 + 4 = 7$
(E)	9	1, 3, 9	$1 + 3 = 4$

The only number from this set that is a perfect number is 6. (Note that the next two perfect number bigger than 28 are 496 and 8128.) ANSWER: (C)

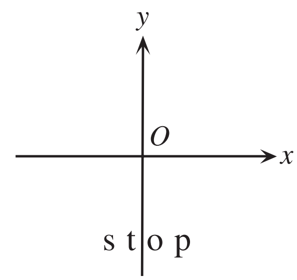
19. Subesha wrote down Davina's phone number in her math binder. Later that day, while correcting her homework, Subesha accidentally erased the last two digits of the phone number, leaving 893-44_ _. Subesha tries to call Davina by dialing phone numbers starting with 893-44. What is the least number of phone calls that she has to make to be guaranteed to reach Davina's house?

(**A**) 100 (**B**) 90 (**C**) 10 (**D**) 1000 (**E**) 20

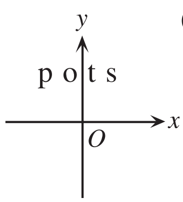
Solution

Davina could have a telephone number between and including 893-4400 and 893-4499. Since there are 100 numbers between and including these two numbers, this is precisely the number of calls that Subesha would have to make to be assured that she would reach Davina's house. An alternate way of seeing this is to realize that Davina's number is of the form 8 9 3 - 4 4 $\underline{a}\,\underline{b}$, where there are 10 possibilities for a and for each of these possibilities, there are 10 possibilities for b. Thus there are 10 ¥ 10 = 100 different possibilities in total. ANSWER: (A)

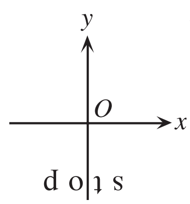
20. The word "stop" starts in the position shown in the diagram to the right. It is then rotated 180∂ clockwise about the origin, O, and this result is then reflected in the x-axis. Which of the following represents the final image?



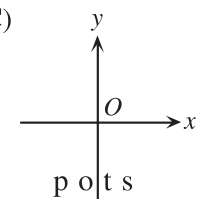
(**A**)



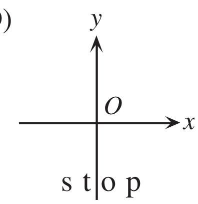
(**B**)



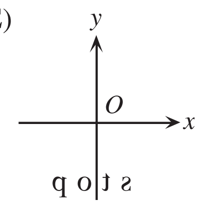
(**C**)



(**D**)

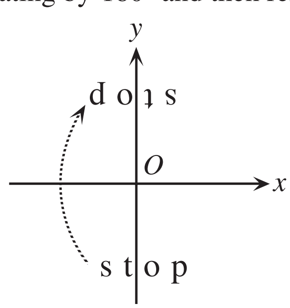


(**E**)

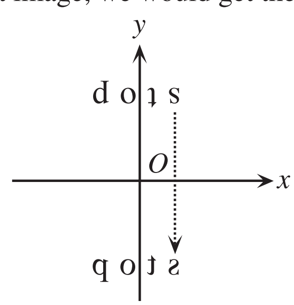


Solution

If we start by rotating by 180° and then reflecting that image, we would get the following:



Rotation of 180°



Reflection in x-axis

ANSWER: (E)

Part C

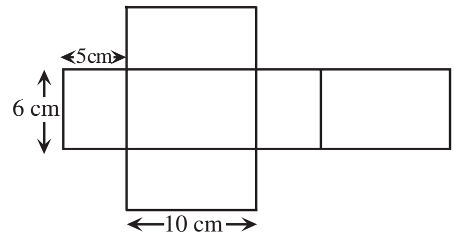
21. Five people are in a room for a meeting. When the meeting ends, each person shakes hands with each of the other people in the room exactly once. The total number of handshakes that occurs is
(**A**) 5 (**B**) 10 (**C**) 12 (**D**) 15 (**E**) 25

Solution

Each of the five people in the room will shake four others' hands. This gives us 20 handshakes, except each handshake is counted twice (Person X shakes Person Y's hand *and* Person Y shakes Person X's hand), so we have to divide the total by 2, to obtain 10 handshakes in total. ANSWER: (B)

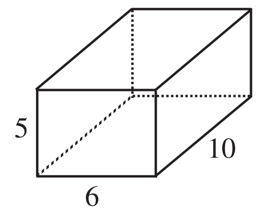
22. The figure shown can be folded along the lines to form a rectangular prism. The surface area of the rectangular prism, in cm^2, is

(**A**) 312 (**B**) 300 (**C**) 280
(**D**) 84 (**E**) 600



Solution

The required surface area is $2(5 \times 6 + 5 \times 10 + 6 \times 10) = 280\ cm^2$.
Alternatively, if we fold the net into a rectangular box, we would obtain the following diagram. From this we can see that the faces of the box are two rectangles of area $30\ cm^2$, two rectangles of area $50\ cm^2$, and two rectangles of area $60\ cm^2$. This gives a total surface area of $280\ cm^2$.



Answer: (C)

23. Mark has a bag that contains 3 black marbles, 6 gold marbles, 2 purple marbles, and 6 red marbles. Mark adds a number of white marbles to the bag and tells Susan if she now draws a marble at random from the bag, the probability of it being black or gold is $\frac{3}{7}$. The number of white marbles that Mark adds to the bag is

(**A**) 5 (**B**) 2 (**C**) 6 (**D**) 4 (**E**) 3

Solution

Since the probability of selecting a black or gold marble is $\frac{3}{7}$, this implies that the total number of marbles in the bag is a multiple of 7. That is to say, there are possibly 7, 14, 21, 28, etc. marbles in the bag. The only acceptable number of marbles in the bag is 21, since there are 9 marbles in total which are black or gold, and $\frac{9}{21} = \frac{3}{7}$. If there are 21 marbles in the bag, this means that 4 marbles must have been added, since there are 17 already accounted for.
Alternatively, we could say that the number of white marbles in the bag was w (an unknown number), and form the equation

$$\frac{6+3}{17+w} = \frac{3}{7}$$

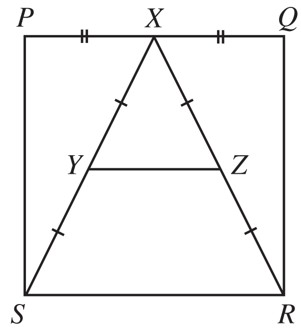
$$\frac{9}{17+w} = \frac{3}{7}$$

$$\frac{9}{17+w} = \frac{9}{21}, \text{ changing to a numerator of 9.}$$

Thus, $17 + w = 21$ or $w = 4$, and so the number of white marbles is 4. Answer: (D)

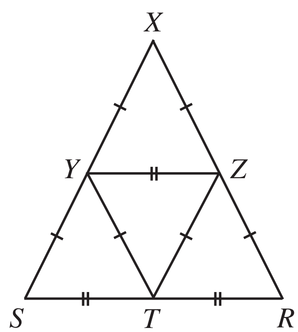
24. *PQRS* is a square with side length 8. *X* is the midpoint of side PQ, and *Y* and *Z* are the midpoints of *XS* and *XR*, respectively, as shown. The area of trapezoid *YZRS* is

(**A**) 24 (**B**) 16 (**C**) 20
(**D**) 28 (**E**) 32



Solution

If *PQRS* is a square with side length 8, it must have an area of 64 square units. The area of $\triangle XSR$ is thus $\frac{1}{2}(8)(8) = 32$. If we take the point *T* to be the midpoint of *SR* and join *Y* and *Z* to *T*, we would have the following diagram.



Each of the four smaller triangles contained within $\triangle XSR$ has an equal area, which is therefore $\frac{1}{4}(32) = 8$. Since the area of trapezoid *YZRS* is made up of three of these triangles, it has an area of $3 \times 8 = 24$.

ANSWER: (A)

25. Each of the integers 226 and 318 have digits whose product is 24. How many three-digit positive integers have digits whose product is 24?

(**A**) 4 (**B**) 18 (**C**) 24 (**D**) 12 (**E**) 21

Solution

First, we determine all of the possible ways to write 24 as the product of single-digit numbers.

(i) $24 = 1 \times 4 \times 6$
(ii) $24 = 1 \times 3 \times 8$
(iii) $24 = 2 \times 3 \times 4$
(iv) $24 = 2 \times 2 \times 6$

The cases numbered (i), (ii) and (iii) each give 6 possible arrangements. For example, if we consider $24 = 1 \times 4 \times 6$, the 6 possibilities are then 146, 164, 416, 461, 614, and 641. So for cases (i), (ii) and (iii), we have a total of 18 possibilities.

For the fourth case, there are only 3 possibilities, which are 226, 262 and 622.

In total there are $18 + 3 = 21$ possibilities.

ANSWER: (E)

* * * * *

Part A

1. The value of $\frac{1}{2}+\frac{1}{4}$ is

(**A**) 1 (**B**) $\frac{1}{8}$ (**C**) $\frac{1}{6}$ (**D**) $\frac{2}{6}$ (**E**) $\frac{3}{4}$

Solution
Using a common denominator,
$\frac{1}{2}+\frac{1}{4}=\frac{2}{4}+\frac{1}{4}=\frac{3}{4}$. ANSWER: (E)

2. The expression $6 ¥ 1000+5 ¥ 100+6 ¥ 1$ is equivalent to
(**A**) 656 (**B**) 6506 (**C**) 6056 (**D**) 60 506 (**E**) 6560

Solution
Expanding,
$6 ¥ 1000+5 ¥ 100+6 ¥ 1=6000+500+6=6506$. ANSWER: (B)

3. The value of $3^2-(4 ¥ 2)$ is
(**A**) 4 (**B**) 17 (**C**) 1 (**D**) –2 (**E**) 0

Solution
By order of operations,
$3^2-(4 ¥ 2)=9-(4 ¥ 2)=9-8=1$. ANSWER: (C)

4. An integer is divided by 7 and the remainder is 4. An example of such an integer is
(**A**) 14 (**B**) 15 (**C**) 16 (**D**) 17 (**E**) 18

Solution
Since 14 is a multiple of 7, then 18 (which is 4 more than 14) gives a remainder of 4 when divided by 7. ANSWER: (E)

5. Which of the following expressions is equal to an odd integer?

(**A**) $3(5)+1$ (**B**) $2(3+5)$ (**C**) $3(3+5)$ (**D**) $3+5+1$ (**E**) $\frac{3+5}{2}$

Solution
Evaluating the choices,

(**A**) $3(5)+1=16$ (**B**) $2(3+5)=16$ (**C**) $3(3+5)=24$ (**D**) $3+5+1=9$ (**E**) $\frac{3+5}{2}=4$

Choice **(D)** gives the only odd integer. ANSWER: (D)

6. Qaddama is 6 years older than Jack. Jack is 3 years younger than Doug. If Qaddama is 19 years old, how old is Doug?

(**A**) 17 (**B**) 16 (**C**) 10 (**D**) 18 (**E**) 15

Solution

If Qaddama is 6 years older than Jack and she is 19 years old, then Jack is 13 years old. If Jack is 3 years younger than Doug, then Doug must be 16 years of age. ANSWER: (B)

7. The volume of a rectangular box is 144 cm^3. If its length is 12 cm and its width is 6 cm, what is its height?

(**A**) 126 cm (**B**) 72 cm (**C**) 4 cm (**D**) 8 cm (**E**) 2 cm

Solution

We know that $\text{Volume} = \text{Length} ¥ \text{Width} ¥ \text{Height}$, the volume is 144 cm^3, and $\text{Length} ¥ \text{Width} = 72\text{ cm}^2$. Thus, $144\text{ cm}^3 = 72\text{ cm}^2 ¥ \text{Height}$, or $\text{Height} = 2\text{ cm}$.

ANSWER: (E)

8. In a jar, the ratio of the number of oatmeal cookies to the number of chocolate chip cookies is 5:2. If there are 20 oatmeal cookies, the number of chocolate chip cookies in the jar is

(**A**) 28 (**B**) 50 (**C**) 8 (**D**) 12 (**E**) 18

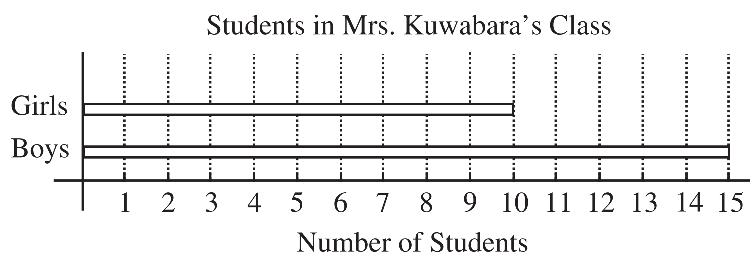
Solution

The ratio 5:2 indicates that there are 5 oatmeal cookies for every 2 chocolate chip cookies. Since there are 20 oatmeal cookies, there are four groups of 5 oatmeal cookies. Thus there are $4 ¥ 2 = 8$ chocolate chip cookies.

Algebraically, we could let x represent the number of chocolate chip cookies. Then $5:2 = 20:x$, or $\frac{5}{2} = \frac{20}{x}$. If we want to write $\frac{5}{2}$ as a fraction with a numerator of 20, we multiply both the numerator and denominator by 4, ie. $\frac{5}{2} = \frac{5 ¥ 4}{2 ¥ 4} = \frac{20}{8}$. Therefore, $x = 8$. ANSWER: (C)

9. The bar graph shows the numbers of boys and girls in Mrs. Kuwabara's class. The percentage of students in the class who are girls is

(**A**) 40% (**B**) 15% (**C**) 25% (**D**) 10% (**E**) 60%



Solution

From the graph, there are 10 girls and 15 boys in the class. Then, there are 25 students in total in the class, so the percentage of girls is $\frac{10}{25} \times 100\% = 40\%$. ANSWER: (A)

10. Which of the following statements is **not** true?
(**A**) A quadrilateral has four sides.
(**B**) The sum of the angles in a triangle is 180°.
(**C**) A rectangle has four 90° angles.
(**D**) A triangle can have two 90° angles.
(**E**) A rectangle is a quadrilateral.

Solution

A quadrilateral has four sides, by definition.
The sum of the angles in a triangle is 180°.
A rectangle has four 90° angles, by definition.
A rectangle is a quadrilateral, since it has four sides.
However, a triangle cannot have two 90°, since its three angles add to 180°, and its third angle cannot be 0°. ANSWER: (D)

Part B

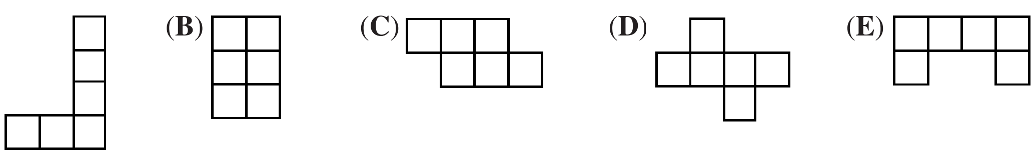
11. A palindrome is a positive integer whose digits are the same when read forwards or backwards. 2002 is a palindrome. What is the smallest number which can be added to 2002 to produce a larger palindrome?
(**A**) 11 (**B**) 110 (**C**) 108 (**D**) 18 (**E**) 1001

Solution

The best way to analyze this problem is by asking the question, "What is the next palindrome bigger than 2002?" Since the required palindrome should be of the form $2aa2$, where the middle two digits (both a) do not equal 0, it must be the number 2112. Thus, the number that must be added to 2002 is $2112 - 2002 = 110$. ANSWER: (B)

12. Which of the following can be folded along the lines to form a cube?

(**A**) (**B**) (**C**) (**D**) (**E**)



Solution

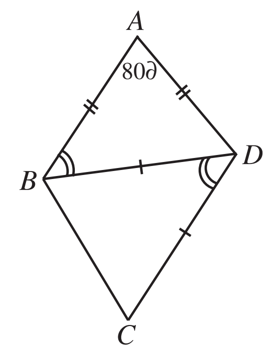
Only choice (**D**) can be folded to form a cube. (Try constructing these nets to check this answer.)
ANSWER: (D)

13. If $a+b=12$, $b+c=16$, and $c=7$, what is the value of a?
(**A**) 1 (**B**) 5 (**C**) 9 (**D**) 7 (**E**) 3

Solution
Since $c=7$ and $b+c=16$, then $b+7=16$, or $b=9$.
Since $b=9$ and $a+b=12$, then $a+9=12$, or $a=3$.

ANSWER: (E)

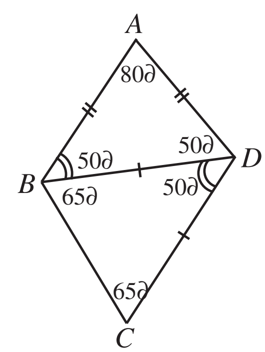
14. In the diagram, $\angle ABD = \angle BDC$ and $\angle DAB = 80^\circ$. Also, $AB = AD$ and $DB = DC$. The measure of $\angle BCD$ is
(**A**) 65° (**B**) 50° (**C**) 80°
(**D**) 60° (**E**) 70°



Solution
Since $\triangle ABD$ is isosceles, then $\angle ABD = \angle ADB$.
Therefore,

$$80^\circ + \angle ABD + \angle ADB = 180^\circ$$
$$2 \times \angle ABD = 100^\circ \quad (\text{since } \angle ABD = \angle ADB)$$
$$\angle ABD = 50^\circ$$

Thus, $\angle BDC = 50^\circ$ as well, since $\angle ABD = \angle BDC$. Since $\triangle BDC$ is also isosceles, then if we repeat a similar calculation to above, we obtain that $\angle BCD = 65^\circ$.



ANSWER: (A)

15. A perfect number is an integer that is equal to the sum of all of its positive divisors, except itself. For example, 28 is a perfect number because $28 = 1+2+4+7+14$. Which of the following is a perfect number?
(**A**) 10 (**B**) 13 (**C**) 6 (**D**) 8 (**E**) 9

Solution
We must check each of the answers:

	Number	Positive divisors	Sum of all positive divisors except itself
(A)	10	1, 2, 5, 10	$1+2+5=8$
(B)	13	1, 13	1
(C)	6	1, 2, 3, 6	$1+2+3=6$
(D)	8	1, 2, 4, 8	$1+2+4=7$
(E)	9	1, 3, 9	$1+3=4$

The only number from this set that is a perfect number is 6. (Note that the next two perfect number bigger than 28 are 496 and 8128.)

Answer: (C)

16. Three pennies are flipped. What is the probability that they all land with heads up?

(**A**) $\frac{1}{8}$ (**B**) $\frac{1}{6}$ (**C**) $\frac{1}{4}$ (**D**) $\frac{1}{3}$ (**E**) $\frac{1}{2}$

Solution

If we toss one penny, the probability that it lands with heads up is $\frac{1}{2}$.

Since we want three heads up, we must multiply these probabilities together. That is, the probability is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$.

Alternatively, we could list all of the possibilities for the 3 pennies, using H to represent heads and T to represent tails:

HHH	THH
HHT	THT
HTH	TTH
HTT	TTT

This means that there are 8 equally likely possibilities, one of which is the desired possibility.

Therefore, the probability of three heads coming up is $\frac{1}{8}$. ANSWER: (A)

17. If P is a negative integer, which of the following is always positive?

(**A**) P^2 (**B**) $\frac{1}{P}$ (**C**) $2P$ (**D**) $P-1$ (**E**) P^3

Solution

If we try $P=-1$,

(**A**) $P^2=1$ (**B**) $\frac{1}{P}=-1$ (**C**) $2P=-2$ (**D**) $P-1=-2$ (**E**) $P^3=-1$

and so the only possibility for the correct answer is (**A**). (In fact, P^2 is always greater than or equal to 0, regardless of the choice for P.) ANSWER: (A)

18. When expanded, the number of zeros in 1000^{10} is

(**A**) 13 (**B**) 30 (**C**) 4 (**D**) 10 (**E**) 1000

Solution

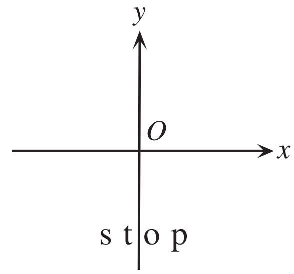
Using exponent laws,

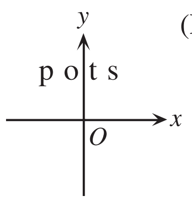
$$1000^{10}=\left(10^3\right)^{10}=10^{3\times 10}=10^{30}$$

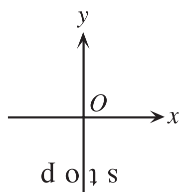
so if we were to write the number out in full, there should be 30 zeros.

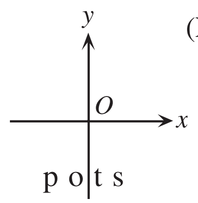
ANSWER: (B)

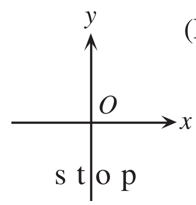
19. The word “stop” starts in the position shown in the diagram to the right. It is then rotated 180∂ clockwise about the origin, O, and this result is then reflected in the x-axis. Which of the following represents the final image?

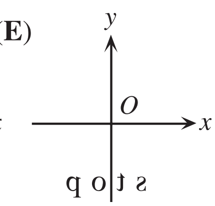


(**A**)


(**B**)


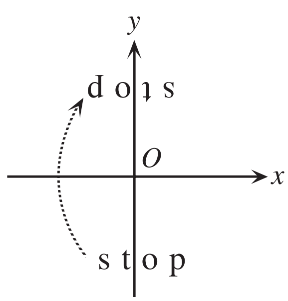
(**C**)


(**D**)


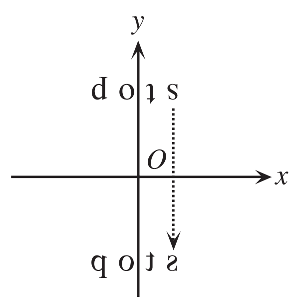
(**E**)


Solution

If we start by rotating by 180° and then reflecting that image, we would get the following:



Rotation of 180°



Reflection in x-axis

Answer: (E)

20. The units digit (that is, the last digit) of 7^{62} is

(**A**) 7 (**B**) 1 (**C**) 3 (**D**) 9 (**E**) 5

Solution

If we write out the first few powers of 7,

$$7^1 = 7,\ 7^2 = 49,\ 7^3 = 343,\ 7^4 = 2401,\ 7^5 = 16\,807, \ldots$$

we can see that the units digit follows the pattern 7, 9, 3, 1, 7, 9, 3, 1, 7, … (That is to say, the units digit of a product depends only on the units digits of the numbers being multiplied together. This tells us that we only need to look at the units digit of the previous power to determine the units digit of a given power.)

So the pattern 7, 9, 3, 1, repeats in blocks of four. Since 60 is a multiple of 4, this means that 7^{60} has a units digit of 1, and so 7^{62} has a units digit of 9.

Answer: (D)

Part C

21. A rectangle has sides of integer length (when measured in cm) and an area of $36\ \text{cm}^2$. What is the maximum possible perimeter of the rectangle?
(**A**) 72 cm (**B**) 80 cm (**C**) 26 cm (**D**) 74 cm (**E**) 48 cm

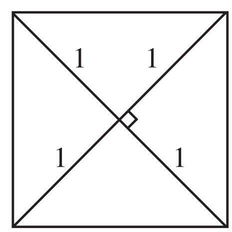
Solution
Since the area is $36\ \text{cm}^2$ and the sides have integer length, then we make a table of the possibilities:

Side lengths	Perimeter
1, 36	$2(1+36)=74$
2, 18	$2(2+18)=40$
3, 12	$2(3+12)=30$
4, 9	$2(4+9)=26$
6, 6	$2(6+6)=24$

So the maximum possible perimeter is 74 cm. Answer: (D)

22. If each diagonal of a square has length 2, then the area of the square is
(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

Solution
We draw the square and its two diagonals.
The diagonals of a square cut each other into two equal parts, and intersect at right angles. So we can decompose the square into 4 identical triangles with base 1 and height 1. So the area of the square is $4\left[\frac{1}{2}(1)(1)\right]=4\left[\frac{1}{2}\right]=2$.



Answer: (B)

23. A map is drawn to a scale of 1:10 000. On the map, the Gauss Forest occupies a rectangular region measuring 10 cm by 100 cm. What is the actual area of the Gauss Forest, in km^2?
(**A**) 100 (**B**) 1 000 000 (**C**) 1000 (**D**) 1 (**E**) 10

Solution
The actual lengths of the sides of the Gauss Forest are 10 000 times the lengths of the sides on the map. So the one side has length

$$10\ 000 \text{¥} 10\ \text{cm} = 100\ 000\ \text{cm} = 1000\ \text{m} = 1\ \text{km}$$

and the other side has length

$$10\ 000 \text{¥} 100\ \text{cm} = 1\ 000\ 000\ \text{cm} = 10\ 000\ \text{m} = 10\ \text{km}.$$

The actual area of the Gauss Forest is therefore $1\ \text{km} \text{¥} 10\ \text{km} = 10\ \text{km}^2$.

Answer: (E)

24. Veronica has 6 marks on her report card.
The mean of the 6 marks is 74.
The mode of the 6 marks is 76.
The median of the 6 marks is 76.
The lowest mark is 50.
The highest mark is 94.
Only one mark appears twice and no mark appears more than twice.
Assuming all of her marks are integers, the number of possibilities for her second lowest mark is
(**A**) 17 (**B**) 16 (**C**) 25 (**D**) 18 (**E**) 24

Solution

Since the mode of Veronica's 6 marks is 76, and only one mark appears more than once (and no marks appear more than twice), then two of the marks must be 76. This tells us that four of her marks were 50, 76, 76, 94.

Since the median of her marks is 76 and she has six marks in total (that is, an *even* number of marks), then the two marks of 76 must be 3rd and 4th when the marks are arranged in increasing order.

Let the second lowest mark be M, and the second highest be N. So the second lowest mark M is between (but not equal to) 50 and 76, and the second highest mark N is between (but not equal to) 76 and 94.

We still need to use the fact that the mean of Veronica's marks is 74, so

$$\frac{50+M+76+76+N+94}{6} = 74$$
$$M+N+296 = 444$$
$$M+N = 148$$
$$M = 148 - N \quad (*)$$

We know already that M is one of 51 through 75, but the possibilities for N and the equation (*) restrict these possibilities further.

Since N can be any of 77 through 93, there are exactly 17 possibilities for N. The largest value of M corresponds to $N = 77$ (ie. $M = 71$) and the smallest value for M is when $N = 93$ (ie. $M = 55$). Thus the possibilities for M are 55 through 71, ie. there are 17 possibilities in total for M, the second smallest mark.

ANSWER: (A)

25. Emily has created a jumping game using a straight row of floor tiles that she has numbered 1, 2, 3, 4, … . Starting on tile 2, she jumps along the row, landing on every second tile, and stops on the second last tile in the row. Starting from this tile, she turns and jumps back toward the start, this time landing on every third tile. She stops on tile 1. Finally, she turns again and jumps along the row, landing on every fifth tile. This time, she again stops on the second last tile. The number of tiles in the row could be
(**A**) 39 (**B**) 40 (**C**) 47 (**D**) 49 (**E**) 53

Solution

Since Emily first starts on tile 2 and jumps on every second tile, then she lands only on even numbered tiles. Since she stops on the second last tile, the total number of tiles is odd.

Next, Emily jumps back along the row by 3's and ends on tile 1. So every tile that she lands on this time has a number which is 1 more than a multiple of 3 (eg. 1, 4, 7, etc.) So the second last tile has a number that is 1 more than a multiple of 3. This tells us that the overall number of tiles in the row is 2 more than a multiple of 3.

These two conditions tell us that the total number of tiles cannot be 39, 40 or 49.

Lastly, Emily jumps by 5's along the row starting at 1. This says each tile that she lands on has a number that is 1 more than a multiple of 5. By the same reasoning as above, the total number of tiles in the row is 2 more than a multiple of 5.

Of the two remaining possibilities (47 and 53), the only one that satisfies this last condition is 47, and so 47 satisfies all 3 of the required conditions.

(Work back through Emily's steps using the fact that she starts with 47 tiles to check that this does work.) ANSWER: (C)

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