Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Fermat Contest (Grade 11)

Wednesday, February 21, 2001

Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

1. Do not open the contest booklet until you are told to do so.

2. You may use rulers, compasses and paper for rough work.

3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.

4. On your response form, print your school name, city/town, and province in the box in the upper right corner.

5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**

6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.

7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 20.

8. Diagrams are *not* drawn to scale. They are intended as aids only.

9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Part A: Each correct answer is worth 5.

1. If $x + 2x + 3x + 4x = 5$, then x equals

   (A) 10   (B) $\frac{1}{2}$ (C) $\frac{5}{4}$ (D) 2   (E) $\frac{5}{9}$

2. If $x = \frac{1}{4}$, which of the following has the largest value?

   (A) x (B) x^2 (C) $\frac{1}{2}x$ (D) $\frac{1}{x}$ (E) $\sqrt{x}$

3. In a school, 30 boys and 20 girls entered the Fermat competition. Certificates were awarded to 10% of the boys and 20% of the girls. Of the students who participated, the percentage that received certificates was

   (A) 14   (B) 15   (C) 16   (D) 30   (E) 50

4. Two rectangles overlap with their common region being a smaller rectangle, as shown. The total area of the shaded region is

   (A) 45   (B) 70   (C) 52
   (D) 79   (E) 73
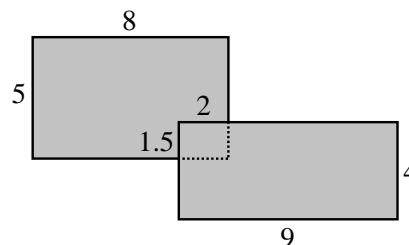
5. In $\triangle ABC$, $\angle A = 3 \angle B$ and $\angle B = 2 \angle C$. The measure of $\angle B$ is
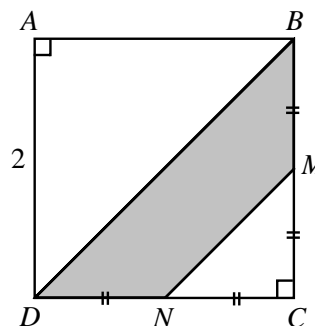
   (A) 10°   (B) 20°   (C) 30°   (D) 40°   (E) 60°

6. Pat gives half of his marbles to his best friend and then a third of those remaining to his sister. If his sister receives 9 marbles, then the number Pat keeps is

   (A) 27   (B) 54   (C) 18   (D) 36   (E) 9

7. In the diagram, square $ABCD$ has side length 2, with M the midpoint of BC and N the midpoint of CD. The area of the shaded region $BMND$ is

   (A) 1   (B) $2\sqrt{2}$ (C) $\frac{4}{3}$

   (D) $\frac{3}{2}$ (E) $4 - \frac{3}{2}\sqrt{2}$

8. If $\sqrt{5 + 11} - 7 = \sqrt{5 + 11} - \sqrt{x}$, then the value of x is

   (A) 1   (B) 7   (C) -7 (D) 49   (E) 4

9. A bag contains 20 candies: 4 chocolate, 6 mint and 10 butterscotch. Candies are removed randomly from the bag and eaten. What is the minimum number of candies that must be removed to be *certain* that at least two candies of each flavour have been eaten?

(**A**) 6   (**B**) 10   (**C**) 12   (**D**) 16   (**E**) 18

10. When a positive integer N is divided by 60, the remainder is 49. When N is divided by 15, the remainder is

(**A**) 0   (**B**) 3   (**C**) 4   (**D**) 5   (**E**) 8

Part B: Each correct answer is worth 6.

11. The fourth root of 2001200120012001 (that is, $\sqrt[4]{2001200120012001}$) is closest to

(**A**) 2 001   (**B**) 6 700   (**C**) 21 000   (**D**) 12 000   (**E**) 2 100

12. How many integer values of x satisfy $\dfrac{x-1}{3} < \dfrac{5}{7} < \dfrac{x+4}{5}$?

(**A**) 0   (**B**) 1   (**C**) 2   (**D**) 3   (**E**) 4

13. *ABCDEFGH* is a cube having a side length of 2. *P* is the midpoint of *EF*, as shown. The area of $\triangle APB$ is
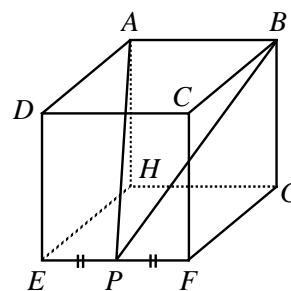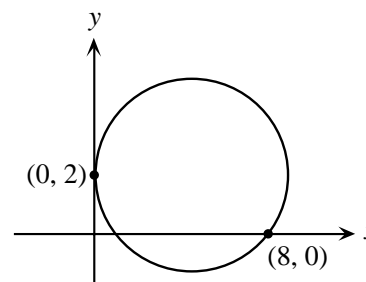
(**A**) $\sqrt{8}$ (**B**) 3   (**C**) $\sqrt{32}$

(**D**) $\sqrt{2}$ (**E**) 6

14. The last digit (that is, the units digit) of $(2002)^{2002}$ is

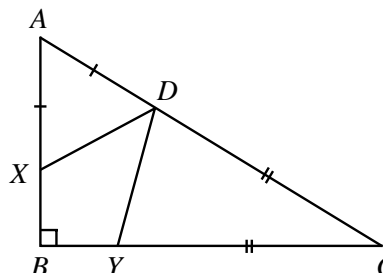(**A**) 4   (**B**) 2   (**C**) 8   (**D**) 0   (**E**) 6

15. A circle is tangent to the *y*-axis at $(0, 2)$, and the larger of its *x*-intercepts is 8. The radius of the circle is

(**A**) $\dfrac{9}{2}$ (**B**) $\sqrt{17}$ (**C**) $\dfrac{17}{4}$

(**D**) $\dfrac{15}{4}$ (**E**) $\dfrac{\sqrt{17}}{2}$

16. In right triangle *ABC*, $AX = AD$ and $CY = CD$, as shown. The measure of $\angle XDY$ is

(**A**) 35°   (**B**) 40°   (**C**) 45°

(**D**) 50°   (**E**) not determined by this information

17. Three different numbers are chosen such that when each of the numbers is added to the average of the remaining two, the numbers 65, 69 and 76 result. The average of the three original numbers is

(A) 34   (B) 35   (C) 36   (D) 37   (E) 38

18. In the diagram, the two smaller circles have equal radii. Each of the three circles is tangent to the other two circles, and each is also tangent to sides of the rectangle. If the width of the rectangle is 4, then its length is
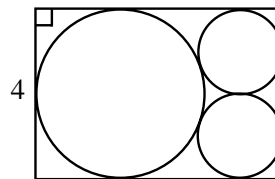
(A) $2 + \sqrt{8}$ (B) $3 + \sqrt{8}$ (C) $3 + \sqrt{10}$

(D) $\sqrt{32}$ (E) $4 + \sqrt{3}$

19. Cindy leaves school at the same time every day. If she cycles at 20 km/h, she arrives home at 4:30 in the afternoon. If she cycles at 10 km/h, she arrives home at 5:15 in the afternoon. At what speed, in km/h, must she cycle to arrive home at 5:00 in the afternoon?
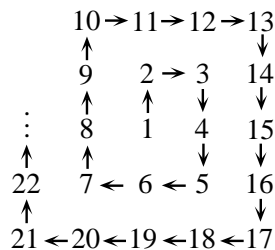
(A) $16\frac{2}{3}$ (B) 15   (C) $13\frac{1}{3}$ (D) 12   (E) $18\frac{3}{4}$

20. Point P is on the line $y = 5x + 3$. The coordinates of point Q are $(3, -2)$. If M is the midpoint of PQ, then M must lie on the line

(A) $y = \frac{5}{2}x - \frac{7}{2}$ (B) $y = 5x + 1$ (C) $y = -\frac{1}{5}x - \frac{7}{5}$ (D) $y = \frac{5}{2}x + \frac{1}{2}$ (E) $y = 5x - 7$

Part C: Each correct answer is worth 8.

21. A spiral of numbers is created, as shown, starting with 1. If the pattern of the spiral continues, in what configuration will the numbers 399, 400 and 401 appear?

```
10→11→12→13
↑           ↓
9    2 → 3   14
↑    ↑   ↓   ↓
:    8   1   4   15
↑    ↑       ↓   ↓
22   7 ← 6 ← 5   16
↑               ↓
21←20←19←18←17
```

(A) $399 \rightarrow 400 \rightarrow 401$ (B) $401 \leftarrow 400 \leftarrow 399$

(C) 401   (D) 399   (E) $400 \rightarrow 401$
   ↑   ↓   ↑
 400   400   399
   ↑   ↓
 399   401

22. A sealed bottle, which contains water, has been constructed by attaching a cylinder of radius 1 cm to a cylinder of radius 3 cm, as shown in Figure A. When the bottle is right side up, the height of the water inside is 20 cm, as shown in the cross-section of the bottle in Figure B. When the bottle is upside down, the height of the liquid is 28 cm, as shown in Figure C. What is the total height, in cm, of the bottle?
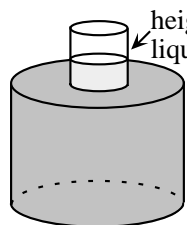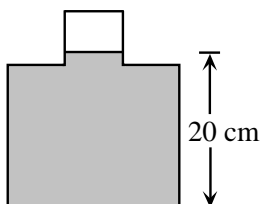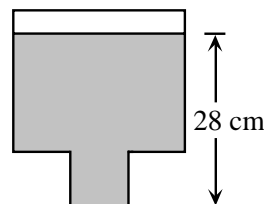
height of liquid

Figure A   Figure B   20 cm   Figure C   28 cm

(A) 29   (B) 30   (C) 31   (D) 32   (E) 48

23. A sequence $t_1, t_2, ..., t_n, ...$ is defined as follows:

$t_1 = 14$

$t_k = 24 - 5t_{k-1}$, for each $k \geq 2$.

For every positive integer n, t_n can be expressed as $t_n = p \cdot q^n + r$, where p, q and r are constants. The value of $p + q + r$ is
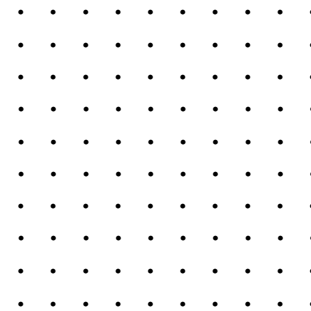
(A) -5 (B) -3 (C) 3   (D) 17   (E) 31

24. The circle with centre A has radius 3 and is tangent to both the positive x-axis and positive y-axis, as shown. Also, the circle with centre B has radius 1 and is tangent to both the positive x-axis and the circle with centre A. The line L is tangent to both circles. The y-intercept of line L is

(A) $3 + 6\sqrt{3}$ (B) $10 + 3\sqrt{2}$ (C) $8\sqrt{3}$

(D) $10 + 2\sqrt{3}$ (E) $9 + 3\sqrt{3}$
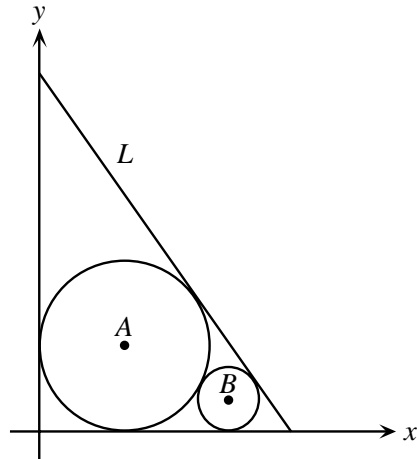
25. A square array of dots with 10 rows and 10 columns is given. Each dot is coloured either blue or red. Whenever two dots of the same colour are adjacent in the same row or column, they are joined by a line segment of the same colour as the dots. If they are adjacent but of different colours, they are then joined by a green line segment. In total, there are 52 red dots. There are 2 red dots at corners with an additional 16 red dots on the edges of the array. The remainder of the red dots are inside the array. There are 98 green line segments. The number of blue line segments is

(A) 36   (B) 37   (C) 38

(D) 39   (E) 40