Part A

1. The value of $987+113-1000$ is
(**A**) 90 (**B**) 10 (**C**) 110 (**D**) 2000 (**E**) 100

Solution
$987+113=1100$
$1100-1000=100$

ANSWER: (E)

2. As a decimal, $\frac{9}{10}+\frac{8}{100}$ is
(**A**) 1.098 (**B**) 0.98 (**C**) 0.098 (**D**) 0.0908 (**E**) 9.8

Solution
Since $\frac{9}{10}=0.9$ and $\frac{8}{100}=0.08$, when we add we get $0.9+0.08=0.98$. ANSWER: (B)

3. What integer is closest in value to $7\times\frac{3}{4}$?
(**A**) 21 (**B**) 9 (**C**) 6 (**D**) 5 (**E**) 1

Solution
$7\times\frac{3}{4}=\frac{21}{4}=5\frac{1}{4}$. The integer closest to $5\frac{1}{4}$ is 5. ANSWER: (D)

4. The value of the expression $5^2-4^2+3^2$ is
(**A**) 20 (**B**) 18 (**C**) 21 (**D**) 10 (**E**) 16

Solution
$5^2=25,\ 4^2=16,\ 3^2=9$
Thus, $5^2-4^2+3^2=25-16+9=18$. ANSWER: (B)

5. When a number is divided by 7, it gives a quotient of 4 with a remainder of 6. What is the number?
(**A**) 17 (**B**) 168 (**C**) 34 (**D**) 31 (**E**) 46

Solution
The required number is $4\times7+6=34$. It is easy to verify this by dividing 34 by 7 which gives a quotient of 4 with remainder 6. ANSWER: (C)

6. In the addition shown, a digit, either the same or different, can be placed in each of the two boxes. What is the sum of the two missing digits?

(**A**) 9 (**B**) 11 (**C**) 13
(**D**) 3 (**E**) 7

$$\begin{array}{r} 8\,6\,3 \\ \square\,9\,1 \\ 7\,\square\,8 \\ \hline 2\,1\,8\,2 \end{array}$$

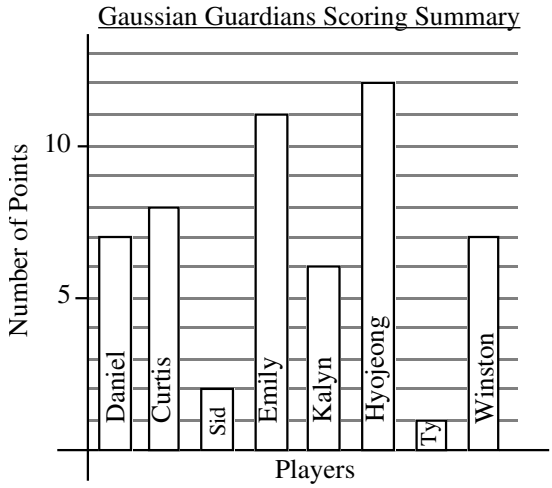
Solution

Adding in the units column gives us, $3+1+8=12$. This means a carry over of 1 into the tens column since $12=1\times10+2$. In the tens column, we have 1 (carried over) $+6+9+\square=18$. The digit that is placed in this box is 2 with a carry over of 1 unit into the hundreds column. Moving to the hundreds column we have, 1 (carried over) $+8+\square+7=21$. The missing digit here is 5. The two missing digits are 2 and 5 giving a sum of 7.

ANSWER: (E)

7. The graph shows the complete scoring summary for the last game played by the eight players on Gaussian Guardians intramural basketball team. The total number of points scored by the Gaussian Guardians was

(**A**) 54 (**B**) 8 (**C**) 12
(**D**) 58 (**E**) 46



Solution

If we list all the players with their points, we would have the following: Daniel (7), Curtis (8), Sid (2), Emily (11), Kalyn (6), Hyojeong (12), Ty (1) and Winston (7).

The total is, $7+8+2+11+6+12+1+7=54$.

ANSWER: (A)

8. If $\frac{1}{2}$ of the number represented by x is 32, what is $2x$?

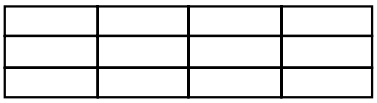
(**A**) 128 (**B**) 64 (**C**) 32 (**D**) 256 (**E**) 16

Solution

If $\frac{1}{2}$ of the number represented by x is 32, then the number x is $2(32)$ or 64 and $2x$ is $2(64)$ or 128.

ANSWER: (A)

9. In the given diagram, all 12 of the small rectangles are the same size. Your task is to completely shade some of the rectangles until $\frac{2}{3}$ of $\frac{3}{4}$ of the diagram is shaded. The number of rectangles you need to shade is



(**A**) 9 (**B**) 3 (**C**) 4
(**D**) 6 (**E**) 8

Solution

Since $\frac{2}{3}\times\frac{3}{4}=\frac{6}{12}=\frac{1}{2}$, the number of shaded rectangles is $\frac{1}{2}\times 12=6$. ANSWER: (D)

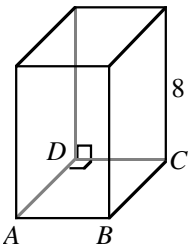
10. The sum of three consecutive integers is 90. What is the largest of the three integers?
(**A**) 28 (**B**) 29 (**C**) 31 (**D**) 32 (**E**) 21

Solution

Since the integers are consecutive, the middle integer is the average of the three integers. The middle integer is $\frac{90}{3}=30$. The integers are 29, 30 and 31. The largest is 31. ANSWER: (C)

Part B

11. A rectangular building block has a square base $ABCD$ as shown. Its height is 8 units. If the block has a volume of 288 cubic units, what is the side length of the base?
(**A**) 6 (**B**) 8 (**C**) 36
(**D**) 10 (**E**) 12



Solution

Since the volume of the rectangular block is 288 cubic units and volume is determined by: (Area of base)(Height), then the area of the base is $\frac{288}{8}=36$. Since we have a square base, it must measure 6×6. The side length of the base is 6 units. ANSWER: (A)

12. A recipe requires 25 mL of butter to be used along with 125 mL of sugar. If 1000 mL of sugar is used, how much butter would be required?
(**A**) 100 mL (**B**) 500 mL (**C**) 200 mL (**D**) 3 litres (**E**) 400 mL

Solution

If 1000 mL of sugar is used, eight times as much sugar would be used as is required by the recipe. We would use 8×25 or 200 mL of butter. ANSWER: (C)

13. Karl had his salary reduced by 10%. He was later promoted and his salary was increased by 10%. If his original salary was $\$20\,000$, what is his present salary?
(**A**) \$16 200 (**B**) \$19 800 (**C**) \$20 000 (**D**) \$20 500 (**E**) \$24 000

Solution
If Karl had his salary reduced by 10%, his new salary was $(0.90)(20\,000) = 18\,000$. If his salary was then increased by 10% his new salary is $(1.10)(18\,000) = 19\,800$. His salary after his 'promotion' is $\$19\,800$.
ANSWER: (B)

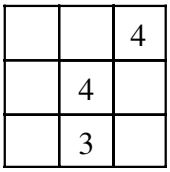
14. The area of a rectangle is 12 square metres. The lengths of the sides, in metres, are whole numbers. The greatest possible perimeter (in metres) is
(**A**) 14 (**B**) 16 (**C**) 12 (**D**) 24 (**E**) 26

Solution
If the rectangle has an area of 12 square metres and its sides are whole numbers then we have only the following possibilities for the width (w), length (l) and corresponding perimeter:

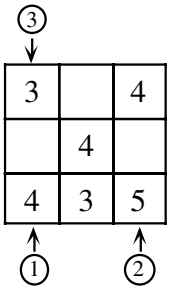
w	l	perimeter
1	12	26
2	6	16
3	4	14

The greatest possible perimeter is 26 m.
ANSWER: (E)

15. In the diagram, all rows, columns and diagonals have the sum 12. What is the sum of the four corner numbers?
(**A**) 14 (**B**) 15 (**C**) 16
(**D**) 17 (**E**) 12



Solution
If we fill in the four corners in the indicated order the sum of the numbers at the corners is $4 + 3 + 4 + 5 = 16$. (This is, of course, not the only way to find the desired number. We could also have started by adding up along the centre column.)

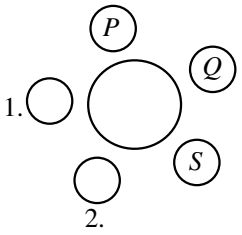


ANSWER: (C)

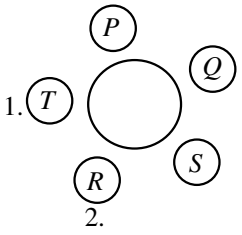
16. Paul, Quincy, Rochelle, Surinder, and Tony are sitting around a table. Quincy sits in the chair between Paul and Surinder. Tony is not beside Surinder. Who is sitting on either side of Tony?
(**A**) Paul and Rochelle (**B**) Quincy and Rochelle (**C**) Paul and Quincy
(**D**) Surinder and Quincy (**E**) Not possible to tell

Solution

If Quincy sits in the chair between Paul and Surinder then these three people would be seated as shown.



Since Tony does not sit beside Surinder then he must sit in the position labelled 1, and Rochelle must sit in the position labelled 2.

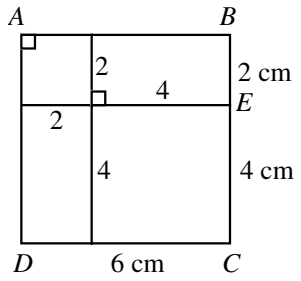


Thus, Tony is seated between Paul and Rochelle as shown in the diagram. ANSWER: (A)

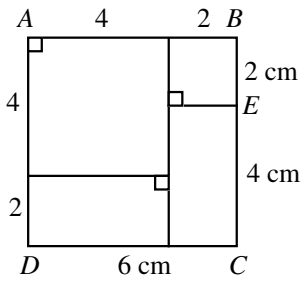
17. $ABCD$ is a square that is made up of two identical rectangles and two squares of area 4 cm^2 and 16 cm^2. What is the area, in cm^2, of the square $ABCD$?
(**A**) 64 (**B**) 49 (**C**) 25 (**D**) 36 (**E**) 20

Solution

One way to draw the required square is shown in the diagram. The smaller square has a side length of 2 cm and the larger a side length of 4 cm. This gives the side length of the larger square to be 6 cm and an area of 36 cm^2.



Note that it is also possible to divide the square up as follows:

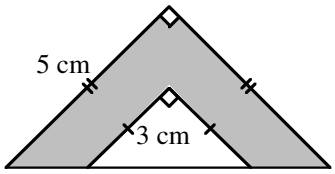


ANSWER: (D)

18. The month of April, 2000, had five Sundays. Three of them fall on even numbered days. The eighth day of this month is a
(**A**) Saturday (**B**) Sunday (**C**) Monday (**D**) Tuesday (**E**) Friday

Solution
Since three of the Sundays fall on even numbered days and two on odd numbered days this implies that the first Sunday of the month must fall on an even numbered day. Note that it is not possible for a Sunday to fall on the 4th day of the month because the 5th Sunday would then have to fall on the 32nd day of the month. The five Sundays will fall on the following days of the calendar: 2, 9, 16, 23, 30. April 8 must be a Saturday. ANSWER: (A)

19. The diagram shows two isosceles right-triangles with sides as marked. What is the area of the shaded region?



(**A**) 4.5 cm^2 (**B**) 8 cm^2 (**C**) 12.5 cm^2
(**D**) 16 cm^2 (**E**) 17 cm^2

Solution
The area of the larger triangle is $\frac{1}{2}(5)(5)$.

The area of the smaller triangle is $\frac{1}{2}(3)(3)$.

The shaded area is, $\frac{1}{2}(5)(5)-\frac{1}{2}(3)(3)$

$$=\frac{1}{2}(25-9)$$

$$=8.$$

Thus the required area is 8 cm^2. ANSWER: (B)

20. A dishonest butcher priced his meat so that meat advertised at \$3.79 per kg was actually sold for \$4.00 per kg. He sold 1 800 kg of meat before being caught and fined \$500. By how much was he ahead or behind where he would have been had he not cheated?
(**A**) \$478 loss (**B**) \$122 loss (**C**) Breaks even (**D**) \$122 gain (**E**) \$478 gain

Solution
The butcher gained \$0.21 on each kg he sold and thus he dishonestly made $(0.21)(1800)=\$378.00$.
After paying the \$500 fine, he would have a loss of $\$500-\$378=\$122$. ANSWER: (B)

Part C

21. In a basketball shooting competition, each competitor shoots ten balls which are numbered from 1 to 10. The number of points earned for each successful shot is equal to the number on the ball. If a competitor misses exactly two shots, which one of the following scores is not possible?
(**A**) 52 (**B**) 44 (**C**) 41 (**D**) 38 (**E**) 35

Solution

If all ten balls scored, a score of 55 is possible.

If ball 1 and 2 is missed the maximum possible score is 52. Similarly, if 9 and 10 are missed, the minimum score is 36. Every score between 36 and 52 is also possible. Of the listed scores, 35 is the only score that is not possible.

ANSWER: (E)

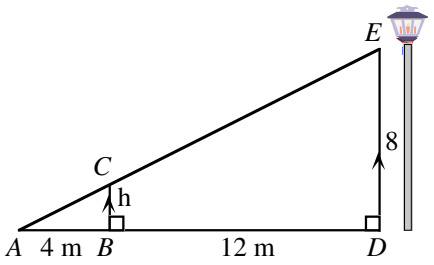
22. Sam is walking in a straight line towards a lamp post which is 8 m high. When he is 12 m away from the lamp post, his shadow is 4 m in length. When he is 8 m from the lamp post, what is the length of his shadow?

(**A**) $1\frac{1}{2}$ m (**B**) 2 m (**C**) $2\frac{1}{2}$ m (**D**) $2\frac{2}{3}$ m (**E**) 3 m

Solution

As Sam approaches the lamp post, we can visualize his position, as shown.

Since ΔABC and ΔADE are similar, the lengths of their corresponding sides are proportional. To determine Sam's height h, we solve $\frac{h}{4} = \frac{8}{16}$, and therefore h = 2 m.



As Sam moves to a position that is 8 m from the lamp post we now have the situation, as shown.

Using similar triangles as before, we can now calculate, L, the length of the shadow.

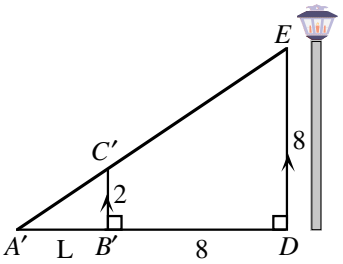
Thus, $\frac{L}{2} = \frac{L+8}{8}$.

Using the property of equivalent fractions, $\frac{L}{2} = \frac{4L}{8} = \frac{L+8}{8}$.

Thus, $4L = L+8$

$3L = 8$

$L = 2\frac{2}{3}$ m



ANSWER: (D)

23. The total area of a set of different squares, arranged from smallest to largest, is 35 km^2. The smallest square has a side length of 500 m. The next larger square has a side length of 1000 m. In the same way, each successive square has its side length increased by 500 m. What is the total number of squares?

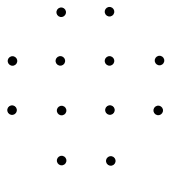
(**A**) 5 (**B**) 6 (**C**) 7 (**D**) 8 (**E**) 9

Solution
We complete the following chart, one row at a time, until 35 appears in the third column.

Number of the square	Length of the square	Area of the square	Cumulative sum of areas
1	0.5 km	0.25 km^2	0.25 km^2
2	1.0 km	1.00 km^2	1.25 km^2
3	1.5 km	2.25 km^2	3.50 km^2
4	2.0 km	4.00 km^2	7.50 km^2
5	2.5 km	6.25 km^2	13.75 km^2
6	3.0 km	9.00 km^2	22.75 km^2
7	3.5 km	12.25 km^2	35.00 km^2

Since there are seven rows, we conclude that there are seven squares. ANSWER: (C)

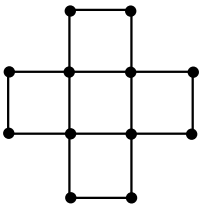
24. Twelve points are marked on a rectangular grid, as shown. How many squares can be formed by joining four of these points?



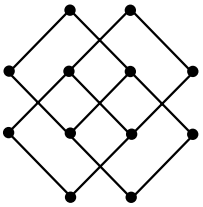
(**A**) 6 (**B**) 7 (**C**) 9
(**D**) 11 (**E**) 13

Solution
In total there are 11 possible squares as shown.

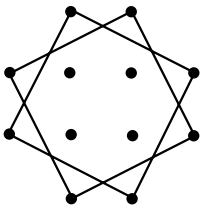
5 small squares:



4 large squares:

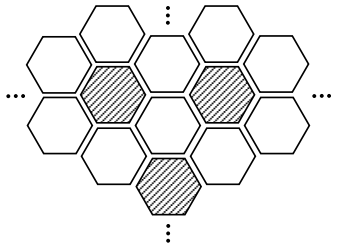


2 that are larger yet:



ANSWER: (D)

25. A square floor is tiled, as partially shown, with a large number of regular hexagonal tiles. The tiles are coloured blue or white. Each blue tile is surrounded by 6 white tiles and each white tile is surrounded by 3 white and 3 blue tiles. Ignoring part tiles, the ratio of the number of blue tiles to the number of white tiles is closest to



(**A**) 1:6 (**B**) 2:3 (**C**) 3:10
(**D**) 1:4 (**E**) 1:2

Solution

Let's start by considering seven tile configurations made up of one blue tile surrounded by six white tiles. If we look just at this tiling only in this way, it appears that there are six times as many white tiles as blue tiles. However, each white tile is adjacent to three different blue tiles. This means that every white tile is part of three different seven tile configurations. Thus, if we count white tiles as simply six times the number counted we will miss the fact that each white tile has been triple counted. Hence the number of white tiles is six times the number of blue tiles divided by three, or twice the number of blue tiles. The ratio of the number of blue tiles to the number of white tiles is 1:2.

ANSWER: (E)

Part A

1. The value of $2^5 + 5$ is

(**A**) 20 (**B**) 37 (**C**) 11 (**D**) 13 (**E**) 21

Solution

$2 \times 2 \times 2 \times 2 \times 2 + 5 = 37$

ANSWER: (B)

2. A number is placed in the box to make the following statement true: $8 + \frac{7}{\square} + \frac{3}{1000} = 8.073$. What is this number?

(**A**) 1000 (**B**) 100 (**C**) 1 (**D**) 10 (**E**) 70

Solution

Since $8.073 = 8 + \frac{0}{10} + \frac{7}{100} + \frac{3}{100}$, the missing number is 100.

ANSWER: (B)

3. The value of $\frac{5+4-3}{5+4+3}$ is

(**A**) -1 (**B**) $\frac{1}{3}$ (**C**) 2 (**D**) $\frac{1}{2}$ (**E**) $-\frac{1}{2}$

Solution

$\frac{5+4-3}{5+4+3} = \frac{6}{12} = \frac{1}{2}$

ANSWER: (D)

4. In the addition shown, a digit, either the same or different, can be placed in each of the two boxes. What is the sum of the two missing digits?

(**A**) 9 (**B**) 11 (**C**) 13
(**D**) 3 (**E**) 7

$$\begin{array}{r} 8\,6\,3 \\ \square\,9\,1 \\ 7\,\square\,8 \\ \hline 2\,1\,8\,2 \end{array}$$

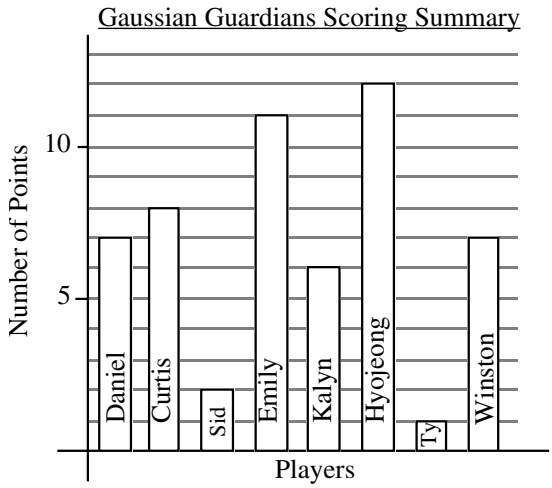
Solution

Adding in the units column gives us, $3 + 1 + 8 = 12$. This means a carry over of 1 into the tens column since $12 = 1 \times 10 + 2$. In the tens column, we have 1 (carried over) $+ 6 + 9 + \square = 18$. The digit that is placed in this box is 2 with a carry over of 1 unit into the hundreds column. Moving to the hundreds column we have, 1 (carried over) $+ 8 + \square + 7 = 21$. The missing digit here is 5. The two missing digits are 2 and 5 giving a sum of 7.

ANSWER: (E)

5. The graph shows the complete scoring summary for the last game played by the eight players on Gaussian Guardians intramural basketball team. The total number of points scored by the Gaussian Guardians was

(**A**) 54 (**B**) 8 (**C**) 12
(**D**) 58 (**E**) 46



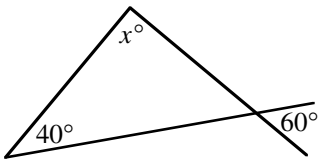
Solution

If we list all the players with their points, we would have the following: Daniel (7), Curtis (8), Sid (2), Emily (11), Kalyn (6), Hyojeong (12), Ty (1) and Winston (7).

The total is, $7+8+2+11+6+12+1+7=54$. ANSWER: (A)

6. In the given diagram, what is the value of x?

(**A**) 20 (**B**) 80 (**C**) 100
(**D**) 120 (**E**) 60

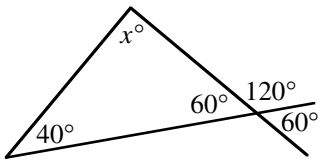


Solution

From the given diagram, we can label the supplementary angle $120°$ and the vertically opposite angle $60°$.

Since the angles in a triangle have a sum of $180°$,

$$x = 180 - (40 + 60)$$
$$x = 80.$$



ANSWER: (B)

7. During the week, the Toronto Stock Exchange made the following gains and losses:

Monday	-150	Thursday	$+182$
Tuesday	$+106$	Friday	-210
Wednesday	-47		

What was the net change for the week?

(**A**) a loss of 119 (**B**) a gain of 119 (**C**) a gain of 91
(**D**) a loss of 91 (**E**) a gain of 695

Solution

$-150+106-47+182-210=-119$

Thus, the net change was a loss of 119 for the week. ANSWER: (A)

8. If $x * y = x + y^2$, then $2 * 3$ equals

(**A**) 8 (**B**) 25 (**C**) 11 (**D**) 13 (**E**) 7

Solution

$2 * 3^2 = 2 + 3^2 = 11$ ANSWER: (C)

9. Of the following five statements, how many are correct?

(i) 20% of $40 = 8$ (ii) $2^3 = 8$ (iii) $7 - 3 \times 2 = 8$ (iv) $3^2 - 1^2 = 8$ (v) $2(6-4)^2 = 8$

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

Solution

(i) True, $\frac{1}{5} \times 40 = 8$

(ii) True, $2^3 = 2 \times 2 \times 2 = 8$

(iii) False, $7 - 3 \times 2 = 7 - 6 = 1$

(iv) True, $9 - 1 = 8$

(v) True, $2(2)^2 = 8$

Only (iii) is false. There are four correct statements. ANSWER: (D)

10. Karl had his salary reduced by 10%. He was later promoted and his salary was increased by 10%. If his original salary was $\$20\,000$, what is his present salary?

(**A**) \$16 200 (**B**) \$19 800 (**C**) \$20 000 (**D**) \$20 500 (**E**) \$24 000

Solution

If Karl had his salary reduced by 10%, his new salary was $(0.90)(20\,000) = 18\,000$. If his salary was then increased by 10% his new salary is $(1.10)(18\,000) = 19\,800$. His salary after his 'promotion' is $\$19\,800$. ANSWER: (B)

Part B

11. Pat planned to place patio stones in a rectangular garden that has dimensions 15 m by 2 m. If each patio stone measures 0.5 m by 0.5 m, how many stones are needed to cover the garden?

(**A**) 240 (**B**) 180 (**C**) 120 (**D**) 60 (**E**) 30

Solution

The garden has an area of $30\ \text{m}^2$.

Each patio stone has an area of $(0.5)(0.5) = 0.25\ \text{m}^2$.

Pat will need $\frac{30}{0.25}$ or 120 patio stones. ANSWER: (C)

12. The prime numbers between 10 and 20 are added together to form the number Q. What is the largest prime divisor of Q?

(**A**) 2 (**B**) 3 (**C**) 5 (**D**) 7 (**E**) 11

Solution

The prime numbers between 10 and 20 are: 11, 13, 17, and 19.

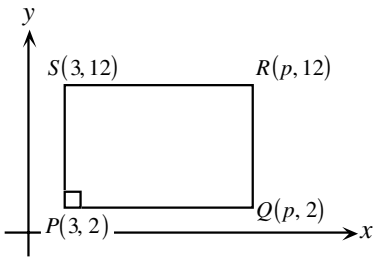
And so, $Q = 11 + 13 + 17 + 19 = 60$.

Since $60 = 2 \times 2 \times 3 \times 5$, the largest prime divisor of Q is 5. ANSWER: (C)

13. The coordinates of the vertices of rectangle $PQRS$ are given in the diagram. The area of rectangle $PQRS$ is 120. The value of p is

(**A**) 10 (**B**) 12 (**C**) 13

(**D**) 14 (**E**) 15



Solution 1

$PS = 12 - 2 = 10$

Since the area of the rectangle is 120,

$$(PS)(PQ) = 120$$
$$(10)(PQ) = 120$$
$$PQ = 12.$$

Therefore, $p = 3 + 12 = 15$.

Solution 2

The dimensions of the rectangle are $(p-3) \times 10$.

Since the area is 120, $10(p-3) = 120$.

Thus, $p - 3 = 12$

or $p = 15$. ANSWER: (E)

14. A set of five different positive integers has an average (arithmetic mean) of 11. What is the largest possible number in this set?

(**A**) 45 (**B**) 40 (**C**) 35 (**D**) 44 (**E**) 46

Solution

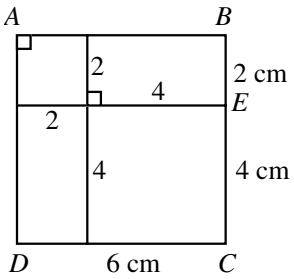
If the set of five different positive integers has an average of 11 the five integers must sum to 5×11 or 55. The four smallest possible integers are 1, 2, 3, and 4. The largest possible integer in the set is $55 - (1 + 2 + 3 + 4) = 45$. ANSWER: (A)

15. *ABCD* is a square that is made up of two identical rectangles and two squares of area 4 cm^2 and 16 cm^2. What is the area, in cm^2, of the square *ABCD*?

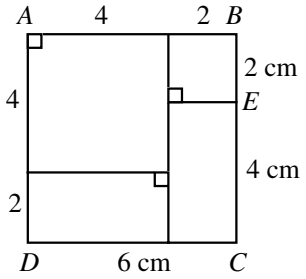
(**A**) 64 (**B**) 49 (**C**) 25 (**D**) 36 (**E**) 20

Solution

One way to draw the required square is shown in the diagram. The smaller square has a side length of 2 cm and the larger a side length of 4 cm. This gives the side length of the larger square to be 6 cm and an area of 36 cm^2.



Note that it is also possible to divide the square up as follows:



ANSWER: (D)

16. Three tenths of our planet Earth is covered with land and the rest is covered with water. Ninety-seven percent of the water is salt water and the rest is fresh water. What percentage of the Earth is covered in fresh water?

(**A**) 20.1% (**B**) 79.9% (**C**) 32.1% (**D**) 2.1% (**E**) 9.6%

Solution

If three tenths of Earth is covered with land then seven tenths or 70% is covered with water. If 97% of this water is salt water then just 3% is fresh water. This implies that 3% of 70% or $(0.03)(0.7) = 0.021 = 2.1\%$ of the Earth is covered in fresh water. ANSWER: (D)

17. In a certain month, three of the Sundays have dates that are even numbers. The tenth day of this month is a

(**A**) Saturday (**B**) Sunday (**C**) Monday (**D**) Tuesday (**E**) Wednesday

Solution
A Sunday must occur during the first three days of any month with five Sundays. Since it is on an even day, it must be on the second day of the month. This implies that the ninth day of the month is also a Sunday, which makes the tenth day a Monday. ANSWER: (C)

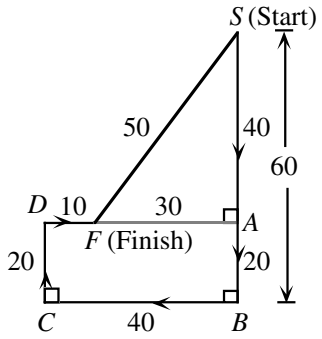
18. Jim drives 60 km south, 40 km west, 20 km north, and 10 km east. What is the distance from his starting point to his finishing point?
(**A**) 30 km (**B**) 50 km (**C**) 40 km (**D**) 70 km (**E**) 35 km

Solution
We can see that Jim's finishing point F is 40 km south and 30 km west of his starting point, S.

By Pythagoras, $AE^2 = 30^2 + 40^2$

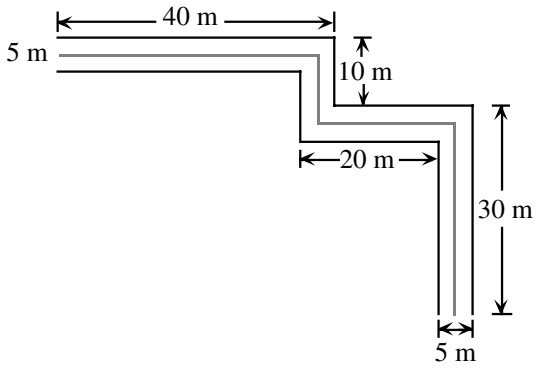
$$AE^2 = 2500$$
$$AE = 50.$$

The distance from his starting point to his end point is 50 km.



ANSWER: (B)

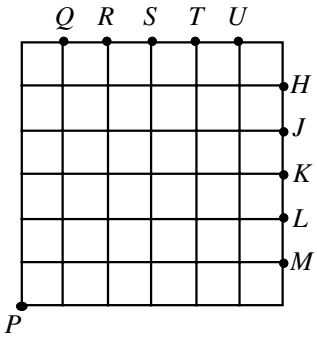
19. A paved pedestrian path is 5 metres wide. A yellow line is painted down the middle. If the edges of the path measure 40 m, 10 m, 20 m, and 30 m, as shown, what is the length of the yellow line?
(**A**) 100 m (**B**) 97.5 m (**C**) 95 m
(**D**) 92.5 m (**E**) 90 m



Solution
Since the path is 5 metres wide, a line in the middle is always 2.5 m from its edges.
Thus the total length is, $37.5 + 10 + 20 + 27.5$
$= 95$ m ANSWER: (C)

20. In the 6 by 6 grid shown, two lines are drawn through point P, dividing the grid into three regions of equal area. These lines will pass through the points

(**A**) M and Q (**B**) L and R (**C**) K and S
(**D**) H and U (**E**) J and T



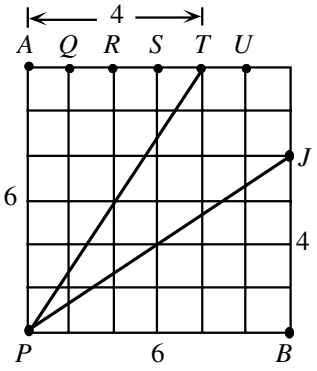
Solution

Label points A and B as shown.

The area of the whole square is 36.

Since the square is divided into three equal areas, each area must be, $\frac{36}{3} = 12$.

The first required point must be one of the points from Q to U. It would have to be a part of a right triangle which would have AP as its height (or its base). Since $AP = 6$ then the base of the triangle would have to be 4 since $\frac{1}{2}(6)(4) = 12$, T is the only point that meets the requirement. In the same way, J also meets the requirement. The required points are thus J and T.



ANSWER: (E)

Part C

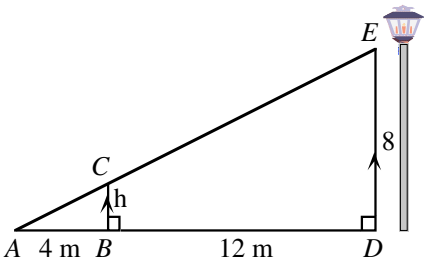
21. Sam is walking in a straight line towards a lamp post which is 8 m high. When he is 12 m away from the lamp post, his shadow is 4 m in length. When he is 8 m from the lamp post, what is the length of his shadow?

(**A**) $1\frac{1}{2}$ m (**B**) 2 m (**C**) $2\frac{1}{2}$ m (**D**) $2\frac{2}{3}$ m (**E**) 3 m

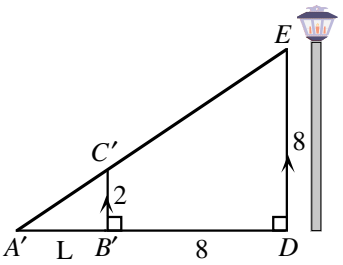
Solution

As Sam approaches the lamp post, we can visualize his position, as shown.

Since ΔABC and ΔADE are similar, the lengths of their corresponding sides are proportional. To determine Sam's height h, we solve $\frac{h}{4} = \frac{8}{16}$, and therefore h = 2 m.



As Sam moves to a position that is 8 m from the lamp post we now have the situation, as shown.
Using similar triangles as before, we can now calculate, L, the length of the shadow.



Thus, $\frac{L}{2} = \frac{L+8}{8}$.

Using the property of equivalent fractions, $\frac{L}{2} = \frac{4L}{8} = \frac{L+8}{8}$.

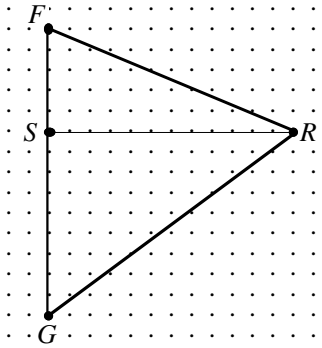
Thus, $4L = L + 8$

$3L = 8$

$L = 2\frac{2}{3}$ m

ANSWER: (D)

22. The homes of Fred (F), Sandy (S), Robert (R), and Guy (G) are marked on the rectangular grid with straight lines joining them. Fred is considering four routes to visit each of his friends:



(i) $F \to R \to S \to G$ (ii) $F \to S \to G \to R$

(iii) $F \to R \to G \to S$ (iv) $F \to S \to R \to G$

If $FS = 5$ km, $SG = 9$ km and $SR = 12$ km, the difference between the longest and the shortest trip (in km) is

(**A**) 8 (**B**) 13 (**C**) 15

(**D**) 2 (**E**) 0

Solution

$FS = 5,\ SR = 12 \Rightarrow FR = 13$. (By Pythagoras, $FR^2 = 5^2 + 12^2 = 169$)

$SG = 9,\ SR = 12 \Rightarrow GR = 15$. (By Pythagoras, $GR^2 = 9^2 + 12^2 = 225$)

(i) $FR + RS + SG = 13 + 12 + 9 = 34$ km

(ii) $FS + SG + GR = 5 + 9 + 15 = 29$ km

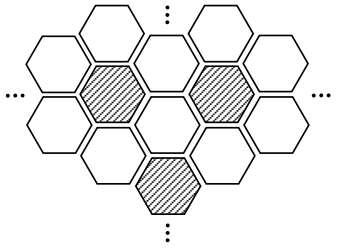
(iii) $FR + RG + GS = 13 + 15 + 9 = 37$ km

(iv) $FS + SR + RG = 5 + 12 + 15 = 32$ km

$37 - 29 = 8$ km is the required distance.

ANSWER: (A)

23. A square floor is tiled, as partially shown, with a large number of regular hexagonal tiles. The tiles are coloured blue or white. Each blue tile is surrounded by 6 white tiles and each white tile is surrounded by 3 white and 3 blue tiles. Ignoring part tiles, the ratio of the number of blue tiles to the number of white tiles is closest to



(**A**) 1:6 (**B**) 2:3 (**C**) 3:10

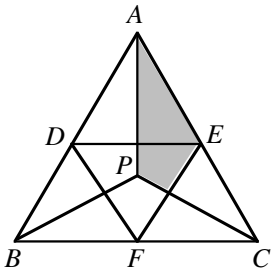
(**D**) 1:4 (**E**) 1:2

Solution

Let's start by considering seven tile configurations made up of one blue tile surrounded by six white tiles. If we look just at this tiling only in this way, it appears that there are six times as many white tiles as blue tiles. However, each white tile is adjacent to three different blue tiles. This means that every white tile is part of three different seven tile configurations. Thus, if we count white tiles as simply six times the number counted we will miss the fact that each white tile has been triple counted. Hence the number of white tiles is six times the number of blue tiles divided by three, or twice the number of blue tiles. The ratio of the number of blue tiles to the number of white tiles is 1:2.

ANSWER: (E)

24. In equilateral triangle ABC, line segments are drawn from a point P to the vertices A, B and C to form three identical triangles. The points D, E and F are the midpoints of the three sides and they are joined as shown in the diagram. What fraction of ΔABC is shaded?



(**A**) $\frac{1}{5}$ (**B**) $\frac{5}{24}$ (**C**) $\frac{1}{4}$

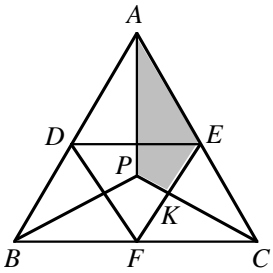
(**D**) $\frac{2}{9}$ (**E**) $\frac{2}{7}$

Solution 1

Since P is a point of symmetry within ΔABC, the line segment CP divides ΔECF into 2 triangles of equal area. That is to say, the area of ΔEKC equals the area of ΔFKC. Since the area of ΔEFC is $\frac{1}{4}$ the area of ΔABC, the area of

$$\Delta EKC = \left(\frac{1}{2} \times \frac{1}{4}\right) \text{ area of } \Delta ABC$$

$$= \frac{1}{8} \text{ (area of } \Delta ABC).$$

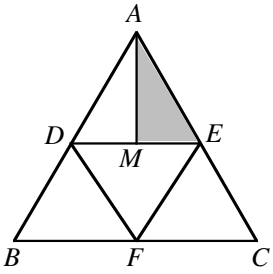


Again since P is a point of symmetry within ΔABC, the area of ΔAPC is $\frac{1}{3}$ the area of ΔABC.

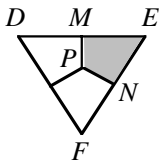
Since the shaded area is the area of ΔAPC – area of ΔKCE, it represents $\left(\frac{1}{3} - \frac{1}{8}\right) \times$ area of $\Delta ABC = \frac{5}{24} \times$ area of ΔABC.

Solution 2

Since D, E and F are the midpoints of the sides, we have four triangles of exactly the same area. That is to say, the areas of ΔADE, ΔDBF, ΔDEF, and ΔEFC are equal. Since ΔAME equals half the area of ΔADE, it represents $\frac{1}{8}$th the area of ΔABC.



Since the figure $MENP$ is one of three identical shapes making up ΔDEF it is one third its area. Since ΔDEF itself is one quarter the area of ΔABC, the figure $MENP$ is $\frac{1}{3}\times\frac{1}{4}$ or $\frac{1}{12}$th the area of ΔABC. Overall, the shaded area is $\frac{1}{8}+\frac{1}{12}=\frac{5}{24}$th the area of ΔABC.



ANSWER: (B)

25. The cookies in a jar contain a total of 1000 chocolate chips. All but one of these cookies contains the same number of chips; it contains one more chip than the others. The number of cookies in the jar is between one dozen and three dozen. What is the sum of the number of cookies in the jar and the number of chips in the cookie with the extra chocolate chip?

(**A**) 65 (**B**) 64 (**C**) 63 (**D**) 66 (**E**) 67

Solution

If we remove the extra chip from the special cookie, all cookies have the same number of chocolate chips for a total of 999 chips. We look at factorizations of 999.

The question states that the number of cookies in the jar is between 12 and 36 so this implies that the only factorization of 999 that works is $(3\times3\times3)(37)$.

Thus the only divisor of 999 between 12 and 36 is 27.

From this, we see that there are 27 cookies.

An ordinary cookie has $\frac{999}{27}=37$ chocolate chips, and the special cookie has 38 chocolate chips.

The required sum is $27+38=65$. ANSWER: (A)