



Canadian Mathematics Competition

An activity of The Centre for Education
in Mathematics and Computing,
University of Waterloo, Waterloo, Ontario

Fermat Contest (Grade 11)

Wednesday, February 23, 2000

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Time: 1 hour

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

Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 20.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2 credits, to a maximum of 20 credits.

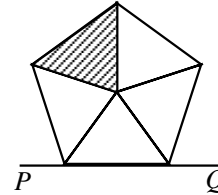
Part A: Each correct answer is worth 5.

- The sum $29 + 12 + 23$ is equal to
(A) 6^2 (B) 4^4 (C) 8^8 (D) 64^0 (E) 2^6
- If the following sequence of five arrows repeats itself continuously, what arrow will be in the 48th position?

(A)  (B)  (C)  (D)  (E) 
- A farmer has 7 cows, 8 sheep and 6 goats. How many more goats should be bought so that half of her animals will be goats?
(A) 18 (B) 15 (C) 21 (D) 9 (E) 6
- The square of 9 is divided by the cube root of 125. What is the remainder?
(A) 6 (B) 3 (C) 16 (D) 2 (E) 1
- The product of 2, 3, 5, and y is equal to its sum. What is the value of y ?
(A) $\frac{1}{3}$ (B) $\frac{10}{31}$ (C) $\frac{10}{29}$ (D) $\frac{3}{10}$ (E) $\frac{10}{3}$
- A student uses a calculator to find an answer but instead of pressing the (x^2) key presses the (\sqrt{x}) key by mistake. The student's answer was 9. What should the answer have been?
(A) 243 (B) 81 (C) 729 (D) 3 (E) 6561
- The sum of the arithmetic series $(-300) + (-297) + (-294) + \dots + 306 + 309$ is
(A) 309 (B) 927 (C) 615 (D) 918 (E) 18
- In a school referendum, $\frac{3}{5}$ of a student body voted 'yes' and 28% voted 'no'. If there were no spoiled ballots, what percentage of the students did not vote?
(A) 72% (B) 40% (C) 32% (D) 12% (E) 88%
- The numbers 6, 14, x , 17, 9, y , 10 have a mean of 13. What is the value of $x + y$?
(A) 20 (B) 21 (C) 23 (D) 25 (E) 35

10. If $x(x(x+1)+2)+3 = x^3 + x^2 + x - 6$ then x is equal to
 (A) 11 (B) -9 (C) -4 or 3 (D) -1 or 0 (E) -2

Part B: Each correct answer is worth 6.

11. When the regular pentagon is reflected in the line PQ , and then rotated *clockwise* 144° about the centre of the pentagon, its position is



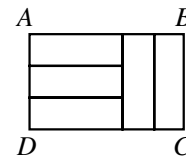
- (A) (B) (C) (D) (E)

12. If the expression $15^6 \times 28^5 \times 55^7$ was evaluated, it would end with a string of consecutive zeros. How many zeros are in this string?

- (A) 10 (B) 18 (C) 26 (D) 13 (E) 5

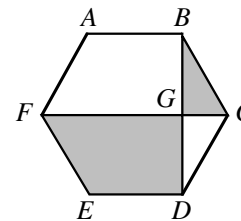
13. Rectangle $ABCD$ is divided into five congruent rectangles as shown. The ratio $AB:BC$ is

- (A) 3:2 (B) 2:1 (C) 5:2
 (D) 5:3 (E) 4:3



14. In the regular hexagon $ABCDEF$, two of the diagonals, FC and BD , intersect at G . The ratio of the area of quadrilateral $FEDG$ to the area of $\triangle BCG$ is

- (A) $3\sqrt{3}:1$ (B) 4:1 (C) 6:1
 (D) $2\sqrt{3}:1$ (E) 5:1



15. In a sequence, every term after the second term is twice the sum of the two preceding terms. The seventh term of the sequence is 8, and the ninth term is 24. What is the eleventh term of the sequence?

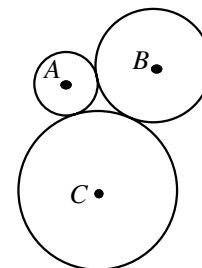
- (A) 160 (B) 304 (C) 28 (D) 56 (E) 64

16. The digits 2, 2, 3, and 5 are randomly arranged to form a four digit number. What is the probability that the sum of the first and last digits is even?

- (A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{6}$ (D) $\frac{1}{2}$ (E) $\frac{2}{3}$

17. Three circles have centres A , B and C with radii 2, 4 and 6 respectively. The circles are tangent to each other as shown. Triangle ABC has

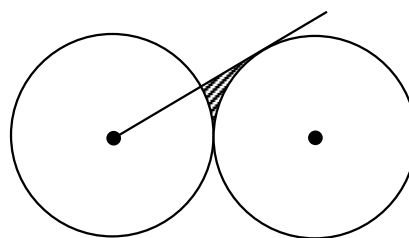
- (A) $\angle A$ obtuse (B) $\angle B = 90^\circ$ (C) $\angle A = 90^\circ$
 (D) all angles acute (E) $\angle B = \angle C$



18. If $P = 3^{2000} + 3^{-2000}$ and $Q = 3^{2000} - 3^{-2000}$ then the value of $P^2 - Q^2$ is
 (A) 3^{4000} (B) 2×3^{-4000} (C) 0 (D) 2×3^{4000} (E) 4
19. An ant walks inside a 18 cm by 150 cm rectangle. The ant's path follows straight lines which always make angles of 45° to the sides of the rectangle. The ant starts from a point X on one of the shorter sides. The first time the ant reaches the opposite side, it arrives at the mid-point. What is the distance, in centimetres, from X to the nearest corner of the rectangle?
 (A) 3 (B) 4 (C) 6 (D) 8 (E) 9
20. Given $a + 2b + 3c + 4d + 5e = k$ and $5a = 4b = 3c = 2d = e$, find the smallest positive integer value for k so that $a, b, c, d,$ and e are all positive integers.
 (A) 87 (B) 522 (C) 10 (D) 120 (E) 60

Part C: Each question is worth 8 credits.

21. Two circles of radius 10 are tangent to each other. A tangent is drawn from the centre of one of the circles to the second circle. To the nearest integer, what is the area of the shaded region?

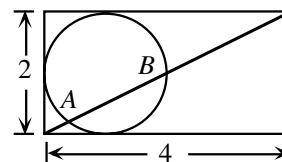


- (A) 6 (B) 7 (C) 8
 (D) 9 (E) 10

22. The left most digit of an integer of length 2000 digits is 3. In this integer, any two consecutive digits must be divisible by 17 or 23. The 2000th digit may be either ' a ' or ' b '. What is the value of $a + b$?

- (A) 3 (B) 7 (C) 4 (D) 10 (E) 17

23. A circle is tangent to three sides of a rectangle having side lengths 2 and 4 as shown. A diagonal of the rectangle intersects the circle at points A and B . The length of AB is

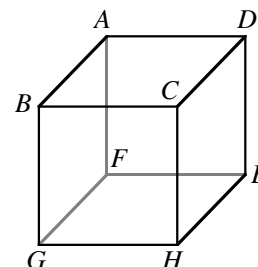


- (A) $\sqrt{5}$ (B) $\frac{4\sqrt{5}}{5}$ (C) $\sqrt{5} - \frac{1}{5}$
 (D) $\sqrt{5} - \frac{1}{6}$ (E) $\frac{5\sqrt{5}}{6}$

24. For the system of equations $x^2 + x^2y^2 + x^2y^4 = 525$ and $x + xy + xy^2 = 35$, the sum of the real y values that satisfy the equations is

- (A) 20 (B) 2 (C) 5 (D) $\frac{55}{2}$ (E) $\frac{5}{2}$

25. The given cube is cut into four pieces by two planes. The first plane is parallel to face $ABCD$ and passes through the midpoint of edge BG . The second plane passes through the midpoints of edges $AB, AD, HE,$ and GH . Determine the ratio of the volumes of the smallest and largest of the four pieces.



- (A) 3:8 (B) 7:24 (C) 7:25
 (D) 7:17 (E) 5:11