Canadian Mathematics Competition

An activity of The Centre for Education
in Mathematics and Computing,
University of Waterloo, Waterloo, Ontario

Fermat Contest (Grade 11)

Wednesday, February 24, 1999

Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
7. Scoring: Each correct answer is worth 5 credits in Part A, 6 credits in Part B, and 8 credits in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2 credits, to a maximum of 20 credits.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2 credits, to a maximum of 20 credits.

Part A: Each question is worth 5 credits.

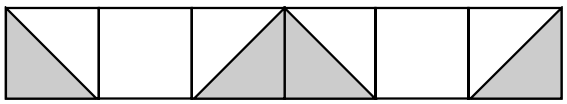
1. The value of $\left(\sqrt{25}-\sqrt{9}\right)^2$ is

   (**A**) 26 (**B**) 16 (**C**) 34 (**D**) 8 (**E**) 4

2. Today is Wednesday. What day of the week will it be 100 days from now?

   (**A**) Monday (**B**) Tuesday (**C**) Thursday (**D**) Friday (**E**) Saturday

3. Six squares are drawn and shaded as shown. What fraction of the total area is shaded?



   (**A**) $\frac{1}{2}$ (**B**) $\frac{1}{3}$ (**C**) $\frac{1}{4}$ (**D**) $\frac{2}{5}$ (**E**) $\frac{2}{3}$

4. Turning a screwdriver 90° will drive a screw 3 mm deeper into a piece of wood. How many complete revolutions are needed to drive the screw 36 mm into the wood?

   (**A**) 3 (**B**) 4 (**C**) 6 (**D**) 9 (**E**) 12

5. A value of x such that $(5-3x)^5=-1$ is

   (**A**) $\frac{4}{3}$ (**B**) 0 (**C**) $\frac{10}{3}$ (**D**) $\frac{5}{3}$ (**E**) 2

6. The number which is 6 less than twice the square of 4 is

   (**A**) -26 (**B**) 10 (**C**) 26 (**D**) 38 (**E**) 58

7. The Partridge family pays each of their five children a weekly allowance. The average allowance for each of the three younger children is \$8. The two older children each receive an average allowance of \$13. The total amount of allowance money paid per week is

   (**A**) \$50 (**B**) \$52.50 (**C**) \$105 (**D**) \$21 (**E**) \$55

8. The time on a digital clock is 5:55. How many minutes will pass before the clock next shows a time with all digits identical?

   (**A**) 71 (**B**) 72 (**C**) 255 (**D**) 316 (**E**) 436

9. In an election, Harold received 60% of the votes and Jacquie received all the rest. If Harold won by 24 votes, how many people voted?

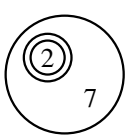
   (**A**) 40 (**B**) 60 (**C**) 72 (**D**) 100 (**E**) 120

10. If x and y are each chosen from the set $\{1, 2, 3, 5, 10\}$, the largest possible value of $\frac{x}{y}+\frac{y}{x}$ is

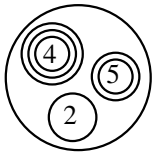
(**A**) 2 (**B**) $12\frac{1}{2}$ (**C**) $10\frac{1}{10}$ (**D**) $2\frac{1}{2}$ (**E**) 20

Part B: Each question is worth 6 credits.

11. In *Circle Land*, the numbers 207 and 4520 are shown in the following way:

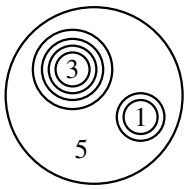


207



4520

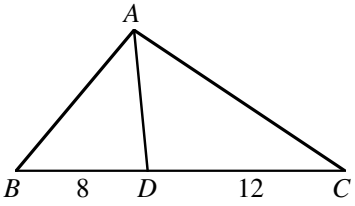
In *Circle Land*, what number does the following diagram represent?



(**A**) 30 105 (**B**) 30 150 (**C**) 3105 (**D**) 3015 (**E**) 315

12. The area of ΔABC is 60 square units. If $BD = 8$ units and $DC = 12$ units, the area (in square units) of ΔABD is

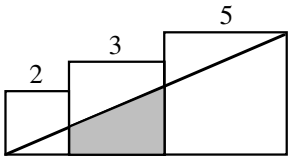
(**A**) 24 (**B**) 40 (**C**) 48
(**D**) 36 (**E**) 6



13. Crippin wrote four tests each with a maximum possible mark of 100. The average mark he obtained on these tests was 88. What is the lowest score he could have achieved on one of these tests?

(**A**) 88 (**B**) 22 (**C**) 52 (**D**) 0 (**E**) 50

14. Three squares have dimensions as indicated in the diagram. What is the area of the shaded quadrilateral?



(**A**) $\frac{21}{4}$ (**B**) $\frac{9}{2}$ (**C**) 5

(**D**) $\frac{15}{4}$ (**E**) $\frac{25}{4}$

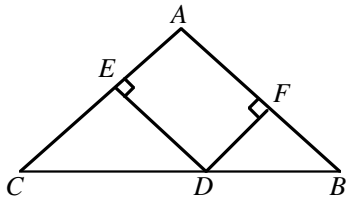
15. If $(a+b+c+d+e+f+g+h+i)^2$ is expanded and simplified, how many different terms are in the final answer?

(**A**) 36 (**B**) 9 (**C**) 45 (**D**) 81 (**E**) 72

16. If $px + 2y = 7$ and $3x + qy = 5$ represent the same straight line, then p equals

(**A**) 5 (**B**) 7 (**C**) 21 (**D**) $\frac{21}{5}$ (**E**) $\frac{10}{7}$

17. In ΔABC, $AC = AB = 25$ and $BC = 40$. D is a point chosen on BC. From D, perpendiculars are drawn to meet AC at E and AB at F. $DE + DF$ equals



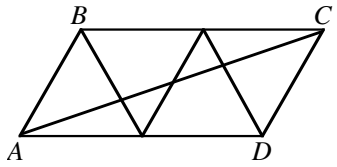
(**A**) 12 (**B**) 35 (**C**) 24

(**D**) 25 (**E**) $\frac{35}{2}\sqrt{2}$

18. The number of solutions (P, Q) of the equation $\frac{P}{Q} - \frac{Q}{P} = \frac{P+Q}{PQ}$, where P and Q are integers from 1 to 9 inclusive, is

(**A**) 1 (**B**) 8 (**C**) 16 (**D**) 72 (**E**) 81

19. Parallelogram $ABCD$ is made up of four equilateral triangles of side length 1. The length of diagonal AC is



(**A**) $\sqrt{5}$ (**B**) $\sqrt{7}$ (**C**) 3
(**D**) $\sqrt{3}$ (**E**) $\sqrt{10}$

20. If $a_1 = \frac{1}{1-x}$, $a_2 = \frac{1}{1-a_1}$, and $a_n = \frac{1}{1-a_{n-1}}$, for $n \geq 2$, $x \neq 1$ and $x \neq 0$, then a_{107} is

(**A**) $\frac{1}{1-x}$ (**B**) x (**C**) $-x$ (**D**) $\frac{x-1}{x}$ (**E**) $\frac{1}{x}$

Part C: Each question is worth 8 credits.

21. How many integers can be expressed as a sum of three distinct numbers if chosen from the set $\{4, 7, 10, 13, ..., 46\}$?

(**A**) 45 (**B**) 37 (**C**) 36 (**D**) 43 (**E**) 42

22. If $x^2 + ax + 48 = (x + y)(x + z)$ and $x^2 - 8x + c = (x + m)(x + n)$, where y, z, m, and n are integers between -50 and 50, then the maximum value of ac is

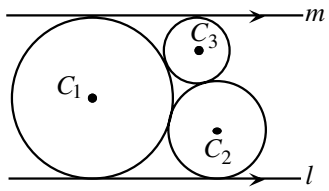
(**A**) 343 (**B**) 126 (**C**) 52 234 (**D**) 784 (**E**) 98 441

23. The sum of all values of x that satisfy the equation $\left(x^2 - 5x + 5\right)^{x^2+4x-60} = 1$ is

(**A**) -4 (**B**) 3 (**C**) 1 (**D**) 5 (**E**) 6

continued ...

24. Two circles C_1 and C_2 touch each other externally and the line l is a common tangent. The line m is parallel to l and touches the two circles C_1 and C_3. The three circles are mutually tangent. If the radius of C_2 is 9 and the radius of C_3 is 4, what is the radius of C_1?



(**A**) 10.4 (**B**) 11 (**C**) $8\sqrt{2}$
(**D**) 12 (**E**) $7\sqrt{3}$

25. Given that n is an integer, for how many values of n is $\dfrac{2n^2-10n-4}{n^2-4n+3}$ an integer?

(**A**) 9 (**B**) 7 (**C**) 6 (**D**) 4 (**E**) 5