2019 Canadian Computing Olympiad
Day 2, Problem 1
Card Scoring

Time Limit: 6 seconds

Problem Description

You have a deck of n cards. Each card has a value: the card values lie between 1 to n, possibly with some repeated values, and possibly with some values never occurring. There is also a special integer k that will be used in calculating a score.

You are playing a game that involves drawing all the cards from the deck one by one. When you draw a card, you may choose to either add it to your hand or discard it. You may also *score* your entire hand at any time. When you score a hand with x cards, you gain $x^{\frac{1}{2}k}$ points and then you discard all cards in that hand. At any point in time, your hand may only contain cards with the same number on them. Given the initial order of the cards in the deck, what is the maximum possible score you can obtain?

Input Specification

The first line of input will contain two space-separated integers k and n. The value k is the value used in the expression $x^{\frac{1}{2}k}$ to compute points ($2 \leq k \leq 4$). The value n is the number of cards in the deck ($1 \leq n \leq 1\,000\,000$). The next n lines contain one integer per line, where the ith of these n lines is the i^{th} card from the top of the deck $(1 \leq i \leq n)$.

For 4 of the 25 marks available, $1 \leq n \leq 20$.

For an additional 1 of the 25 marks available, $1 \leq n \leq 300$, $k = 2$.

For an additional 5 of the 25 marks available, $1 \leq n \leq 300$.

For an additional 3 of the 25 marks available, $1 \leq n \leq 5\,000$.

For an additional 3 of the 25 marks available, $k = 4$.

Output Specification

Output one floating point number, which is the maximum score you can obtain from playing optimally.

If your answer is p and the correct answer is q, then your answer will be considered correct if

$$\frac{|p - q|}{q} \leq 10^{-6}.$$

Sample Input 1

```
3 5
1
2
2
1
1
```

Output for Sample Input 1

```
6.656854249
```

Explanation for Output for Sample Input 1

We know the cards we draw in order are $[1, 2, 2, 1, 1]$ and that discarding a hand of x cards gives us a score of $x^{1.5}$.

The optimal strategy is to draw one card, score that hand, draw two cards, score that hand, and draw two more cards and score that hand. This strategy gives a score of $1^{1.5} + 2^{1.5} + 2^{1.5} \approx 6.656854249$.

Sample Input 2

```
4 5
1
2
2
1
1
```

Output for Sample Input 2

```
9.0
```

Explanation of Output for Sample Input 2

We know the cards we draw in order are $[1, 2, 2, 1, 1]$ and that scoring a hand of x cards gives us a score of x^2.

An optimal strategy is to take all cards with 1, and score them all at the end. This strategy gives a score of $3^2 = 9$. Note that taking the first card and scoring, then taking the next two cards and scoring, and then taking the final two cards and scoring, will also yield $1^2 + 2^2 + 2^2 = 9$.

2019 Canadian Computing Olympiad
Day 2, Problem 2
Marshmallow Molecules

Time Limit: 4 seconds

Problem Description
Hannah is building a structure made of marshmallows and skewers for her chemistry class. The structure will contain N marshmallows, numbered from 1 to N. Some marshmallows will be connected by skewers. Each skewer connects two marshmallows.

Hannah has M requirements for her structure. Each requirement is given as a pair (a_i, b_i), which means that there must be a skewer connecting marshmallow a_i and marshmallow b_i.

To ensure the stability of the structure, the following requirement must also be satisfied: if $a < b < c$, and if there is a skewer connecting marshmallow a to marshmallow b, and if there is a skewer connecting marshmallow a to marshmallow c, then there must also be a skewer connecting marshmallow b to marshmallow c.

Due to austerity measures imposed by the principal's office, skewers are scarce in Hannah's school. Find the minimum number of skewers necessary to satisfy all requirements.

Input Specification
The first line contains two space-separated integers N and M ($1 \leq N, M \leq 10^5$).

The next M lines each contain two space-separated integers, with the i-th line containing a_i and b_i ($1 \leq a_i < b_i \leq N$). All M pairs (a_i, b_i) are distinct.

For 5 of the 25 marks available, $N \leq 100$.

For an additional 5 of the 25 marks available, $N \leq 5000$.

For an additional 5 of the 25 marks available, for all $1 \leq j \leq N$, there is at most one pair (a_i, b_i) such that $b_i = j$.

Output Specification
Output a single integer, the minimum total number of skewers.

Sample Input 1

```
6 4
1 2
1 4
4 6
4 5
```

Output for Sample Input 1

```
6
```

Explanation for Output for Sample Input 1

In addition to those already required, there must be skewers between the pairs of marshmallows $(2, 4)$ and $(5, 6)$.

Sample Input 2

```
7 6
2 3
2 6
2 7
1 3
1 4
1 5
```

Output for Sample Input 2

```
16
```

2019 Canadian Computing Olympiad
Day 2, Problem 3
Bad Codes

Time Limit: 1 second

Problem Description

Your friend has constructed a code that they want to use to send secret messages to you. The messages will only be composed of N different symbols and each symbol will correspond to one binary sequence with at most M bits.

However, you are not sure the code is going to work: there is a chance that a binary sequence can correspond to two (or more) different messages.

For example, if the code was:

$$\begin{aligned} A &\rightarrow 101 \\ B &\rightarrow 10 \\ C &\rightarrow 1 \\ D &\rightarrow 100 \end{aligned}$$

then the binary sequence 101 could be correspond to either A or BC.

Your job is determine the length of the shortest binary sequence that corresponds to two different messages, or determine that there are no binary sequences which correspond to two different messages.

Input Specification

The first line of input will contain two space-separated integers N and M ($1 \leq N, M \leq 50$). The next N lines of input each will have at least one and at most M characters from the set $\{0, 1\}$.

For 4 of the 25 marks available, $N = 4$ and $M \leq 6$.

For an additional 7 of the 25 marks available, each of the binary sequences contains exactly one `1` bit. For example, the sequences `00100` or `1000` would be valid in this case.

Output Specification

Output will be one line long.

If there is a binary sequence that corresponds to two (or more) messages, print the length of the shortest such binary sequence; otherwise, output one line containing `-1`.

Sample Input 1

```
4 3
101
10
1
100
```

Output for Sample Input 1

```
3
```

Explanation of Output for Sample Input 1

This is the sample in the problem description.

Sample Input 2

```
4 4
1011
1000
1111
1001
```

Output for Sample Input 2

```
-1
```

Explanation of Output for Sample Input 2

There is no binary sequence that corresponds to more than one message. Notice that since each code is 4 bits, and none are the same, every encoding of $4k$ bits can be uniquely decoded into k characters.