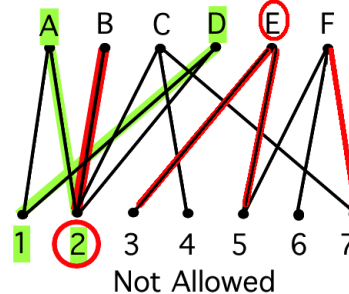
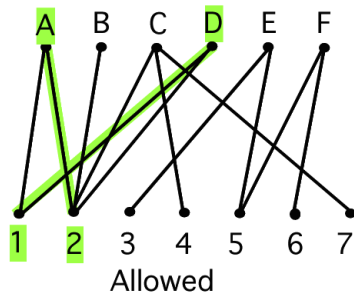
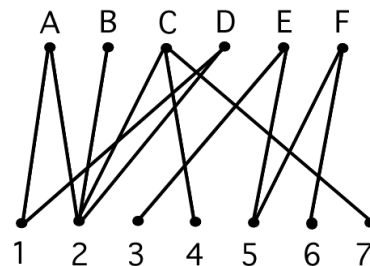
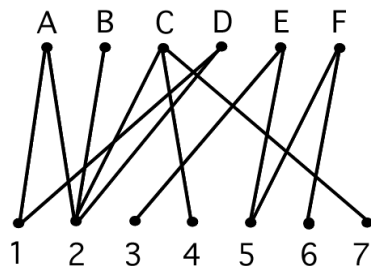
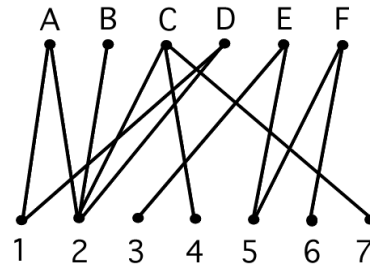
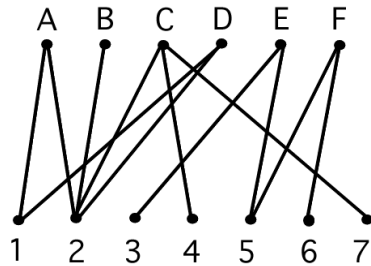


In the graphs below, colour as many edges as possible, without colouring any two edges that touch the same number.



Four copies of the graph are given so that you can try different colourings.

What is the maximum number of edges that can be coloured? How can you be sure?



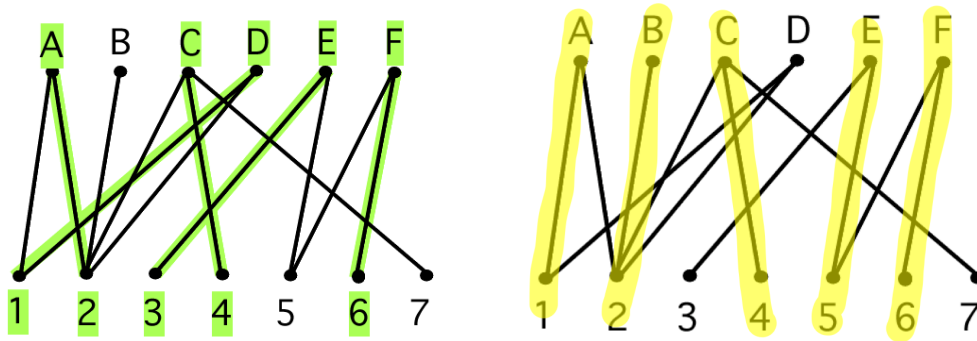
Connection to the Real World

“Over 1,600 Canadians are added to organ wait lists yearly.” –Canadian Transplant Society

Organ transplant organizations keep lists of thousands of potential donors and patients awaiting donations. For a donation to be successful, blood types and other health factors of the patient and donor must be compatible. Mathematicians assist doctors in identifying the most possible matches between patients and donors.

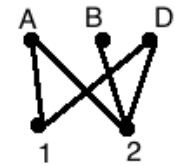
In the graphs above, each letter represents a donor and each number represents a patient. An edge between a letter and number means the corresponding donor and patient are compatible. For example, donor A is compatible with patients 1 and 2, but not with any other patient. **Colouring the maximum number of edges in the graph corresponds to matching as many donors and patients as possible for Organ Donation.**

Solution: Here are two ways to colour 5 edges of the graph.



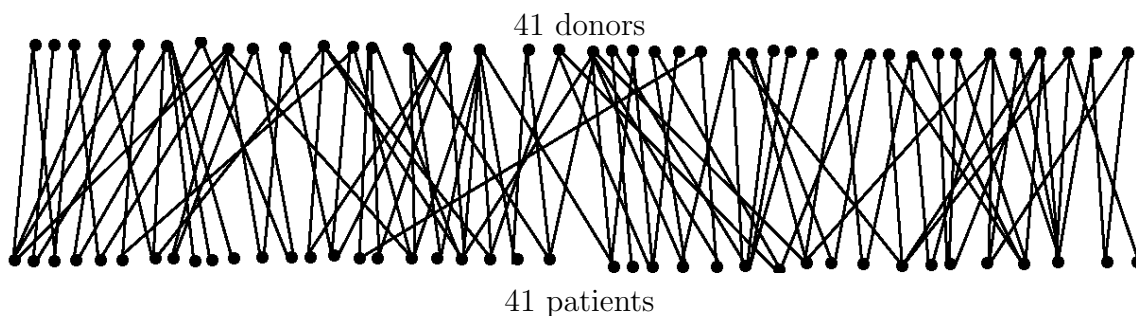
The question of whether more edges can be coloured is an important one. In the context of Organ Donation, colouring one more edge in the graph can result in saving two more lives. We should then convince ourselves that it is not possible to colour 6 edges in this graph.

In order to colour 6 edges in the graph, each donor would need to be matched with a patient. However, donors A, B and D have no compatible patients other than patients 1 and 2. Since there are only two patients compatible with these three donors, one of donors A, B and D must go without donating. This means that the highest number of possible matches is 5.



There is no information given that helps us decide between several different ways of making 5 matches. Factors like the age or health of a patient or their likeliness of finding a match in the future might lead a doctor to prefer one set of matches over another.

The problem becomes more complex even with 41 patients (bottom row) and 41 donors (top row). Imagine a graph with 1600 patients!



Mathematicians use **Graph Theory** techniques and computer algorithms to match the maximum number of compatible teams in a scaled-to-life version of this problem. Ongoing research helps to improve these techniques all the time. Graph theory also has applications as computer networking and traffic flow design.

Related News Articles

<http://www.cbc.ca/news/health/organ-donation-ethics-how-doctors-decide-who-gets-a-transplant-1.2936439>

<http://www.hopkinsmedicine.org/hmn/s09/circling.cfm>

For more Real-World Problems Being Solved by Mathematics, visit <http://www.cemc.uwaterloo.ca/resources/real-world.html>.