Problem of the Week
Grade 9 and 10

Sum Time Soon
Solution

Problem

When fifty consecutive *even* integers are added together, their sum is 3250. Determine the value of the largest number.

Solution 1

Let a represent the smallest number. Since the numbers are even, they increase by 2. So the second number is $(a+2)$, the third is $(a+4)$, the fourth is $(a+6)$, and so on. What does the fiftieth number look like?

A closer look at the numbers reveals that the second number is $(a+1(2))$, the third is $(a+2(2))$, the fourth is $(a+3(2))$, and so on. Following the pattern, the fiftieth number is $(a+49(2)) = a+98$. Then

$$\begin{aligned}
a+(a+2)+(a+4)+(a+6)+\cdots+(a+98) &= 3250\\
50a+2+4+6+\cdots+98 &= 3250\\
50a+2(1+2+3+\cdots+49) &= 3250\\
50a+2\left(\frac{49\times 50}{2}\right) &= 3250, \text{ using the helpful formula.}\\
50a+2450 &= 3250\\
50a &= 800\\
a &= 16\\
a+98 &= 114
\end{aligned}$$

$\therefore$ the largest number is 114.

In the above solution, we looked at patterns. We also used the formula for the sum of the first n positive integers $\frac{(n)(n+1)}{2}$ to find the sum of the positive integers $1+2+3+\cdots+49$. In Solution 2 we will use averages. In Solution 3 we will use arithmetic sequences. This solution is presented last since many students in grade 9 or 10 have not encountered arithmetic sequences yet.

Problem

When fifty consecutive even integers are added together, their sum is 3250. Determine the value of the largest number.

Solution 2

In this solution we will use averages to solve the problem

Let A represent the average of the fifty numbers. The average times the number of integers equals the sum of the integers. Since the sum of the fifty integers is 3 250, then $50A = 3\ 250$ and $A = 65$.

Now the numbers in the sequence are consecutive even integers. The average is odd. It follows that 25 numbers are below the average and 25 numbers are above. We are looking for the 25^{th} even number after the average. In fact we want the 25^{th} even number after the even number 64, the first even number below the average. This number is easily found, $64 + 25(2) = 64 + 50 = 114$.

$\therefore$ the largest number is 114.

In Solution 3, on the next page, arithmetic sequences are used.

Problem

When fifty consecutive even integers are added together, their sum is 3250. Determine the value of the largest number.

Solution 3

In this solution we will use arithmetic sequence formulas to solve the problem. An *arithmetic sequence* is a sequence in which each term after the first is obtained from the previous term by adding a constant. The general term, t_n, of an arithmetic sequence is $t_n = a + (n-1)d$ where a is the first term, d is the difference between consecutive terms and n is the number of terms. The sum of n terms of an arithmetic sequence, S_n, can be found using the formula $S_n = \frac{n}{2}(2a + (n-1)d)$. a, d, and n are the same variables used in the general term formula.

Let a represent the first term in the sequence. Since the numbers in the sequence are consecutive even integers, the numbers go up by two. Therefore $d = 2$. Since there are 50 terms in the sequence, $n = 50$. The sum of the fifty numbers in the sequence is 3250 so $S_{50} = 3250$.

$$\begin{aligned} S_n &= \frac{n}{2}(2a + (n-1)d) \\ 3250 &= \frac{50}{2}(2a + (50-1)(2)) \\ 3250 &= 25(2a + 49(2)) \\ \text{Dividing by 25,} \quad 130 &= 2a + 98 \\ 32 &= 2a \\ 16 &= a \end{aligned}$$

Since we want the largest number in the sequence, we are looking for the fiftieth term.

$$\begin{aligned} \text{Using} \quad t_n &= a + (n-1)d, \quad \text{with } a = 16, d = 2, n = 50 \\ t_{50} &= 16 + 49(2) \\ t_{50} &= 114 \end{aligned}$$

$\therefore$ the largest number is 114.