



## Problem of the Week

### Grade 9 and 10

### An Average Day For Bea

#### Solution

#### Problem

Bea Student has six marks on her report card. The average of her first and second marks is 71%. The average of her second and third marks is 75%. The average of her third and fourth marks is 66%. The average of her fourth and fifth marks is 68%. The average of her fifth and sixth marks is 82%.

Determine (a) Bea's overall average, and (b) the average of Bea's first and sixth marks.

#### Solution

Let  $a, b, c, d, e, f$  represent Bea's six report cards marks.

The average of her first and second marks is 71, so  $\frac{a+b}{2} = 71$ . Multiplying by 2,  $a + b = 142$ . (1)

The average of her second and third marks is 75, so  $\frac{b+c}{2} = 75$ , leading to  $b + c = 150$ . (2)

The average of her third and fourth marks is 66, so  $\frac{c+d}{2} = 66$ , leading to  $c + d = 132$ . (3)

The average of her fourth and fifth marks is 68, so  $\frac{d+e}{2} = 68$ , leading to  $d + e = 136$ . (4)

The average of her fifth and sixth marks is 82, so  $\frac{e+f}{2} = 82$ , leading to  $e + f = 164$ . (5)

- (a) To find Bea's overall average we must find the sum  $a + b + c + d + e + f$  and divide by 6. If we add equations (1), (3) and (5) we obtain the required sum.

$$\begin{aligned}(a + b) + (c + d) + (e + f) &= 142 + 132 + 164 \\ a + b + c + d + e + f &= 438 \\ \frac{a + b + c + d + e + f}{6} &= 73\end{aligned}$$

$\therefore$  Bea's overall average is 73.

- (b) To find the average of Bea's first and sixth marks, we must find the sum  $a + f$  and divide by 6. We will add equations (1), (2), (3), (4) and (5).

$$\begin{aligned}(a + b) + (b + c) + (c + d) + (d + e) + (e + f) &= 142 + 150 + 132 + 136 + 164 \\ a + 2b + 2c + 2d + 2e + f &= 724 \\ a + 2(b + c) + 2(d + e) + f &= 724 \\ a + 2(150) + 2(136) + f &= 724, \text{ substituting from (2) and (4)} \\ a + 300 + 272 + f &= 724 \\ a + f &= 152 \\ \frac{a + f}{2} &= 76\end{aligned}$$

$\therefore$  the average of Bea's first and sixth marks is 76%.

