Problem of the Week
Grade 9 and 10

What Was That Number?
Solution

Problem

Three different numbers, a, b, and c, are written in order smallest to largest such that $a < b < c$. When the numbers are added in pairs the sums $2\ 989, 3\ 461$ and $3\ 550$ are obtained. Determine the value of the largest number.

Solution

The smallest sum is created by adding the smallest two numbers together. Therefore, $a + b = 2\ 989$. The largest sum is created by adding the largest two numbers together. Therefore, $b + c = 3\ 550$. The only possible sum left to calculate is $a + c$ so $a + c = 3\ 461$.

At this point we could solve a system of equations involving three equations and three unknowns. However, there is a simpler approach. We will add equations $a + b = 2\ 989$, $a + c = 3\ 461$ and $b + c = 3\ 550$ together.

$$\begin{aligned}(a+b)+(a+c)+(b+c) &= 2\ 989 + 3\ 461 + 3\ 550\\ 2a+2b+2c &= 10\ 000\\ 2(a+b+c) &= 10\ 000\\ a+b+c &= 5\ 000\end{aligned}$$

So now we have the sum of the three numbers is 5 000. But the sum of the first two numbers $a + b$ is 2 989. We can subtract to obtain the largest number.

$$\begin{aligned}(a+b+c)-(a+b) &= 5\ 000 - 2\ 989\\ a+b+c-a-b &= 2\ 011\\ c &= 2\ 011\end{aligned}$$

The largest number is 2 011. It is quite straight forward to determine that the smallest number is 1 450 and the middle number is 1 539.