

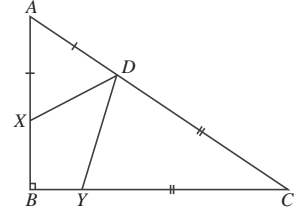


Problem of the Week Grade 9 and 10

A Matter of Degrees Solution

Problem

In right $\triangle ABC$, X is on AB , Y is on BC and D is on AC such that $AX = AD$ and $CY = CD$. Determine the measure of $\angle XDY$.



Solution

Let $\angle CDY = x$. Since $CD = CY$, $\triangle CDY$ is isosceles and $\angle CYD = \angle CDY = x$.

In a triangle, the angles sum to 180° .

$$\text{In } \triangle CDY, \angle C + \angle CDY + \angle CYD = 180^\circ$$

$$\angle C + x + x = 180^\circ$$

$$\angle C = 180^\circ - 2x$$

$$\text{In } \triangle ABC, \angle A + \angle B + \angle C = 180^\circ$$

$$\angle A + 90^\circ + 180^\circ - 2x = 180^\circ$$

$$\angle A = 2x - 90^\circ$$

Let $\angle ADX = y$. Since $AD = AX$, $\triangle ADX$ is isosceles and $\angle AXD = \angle ADX = y$.

$$\text{In } \triangle ADX, \angle ADX + \angle AXD + \angle A = 180^\circ$$

$$y + y + 2x - 90^\circ = 180^\circ$$

$$2y = 270^\circ - 2x$$

$$y = 135^\circ - x$$

ADC forms a straight line so $\angle ADC = 180^\circ$.

$$\text{So, } \angle ADX + \angle XDY + \angle CDY = 180^\circ$$

$$y + \angle XDY + x = 180^\circ$$

$$135^\circ - x + \angle XDY + x = 180^\circ$$

$$\angle XDY = 180^\circ - 135^\circ$$

$$\therefore \angle XDY = 45^\circ$$

