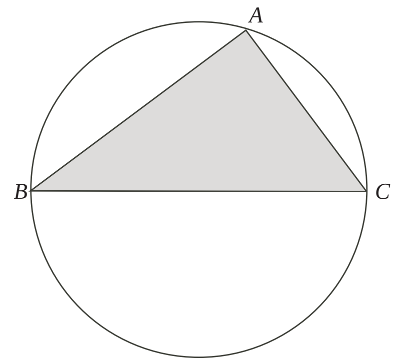
Problem of the Week
Grade 9 and 10

An Unshady Area
Solution

Problem

$\triangle ABC$ is inscribed in a circle with vertices B and C located at the endpoints of a diameter and the third vertex, A, on the circumference of the circle so that $AB = 16$ cm, $AC = 12$ cm, and $BC = 20$ cm.



Determine the area of the unshaded region of the circle. Express the area as an exact answer involving π. Then state the area correct to the nearest hundredth cm^2.

Solution

In $\triangle ABC$, $AB^2 + AC^2 = 16^2 + 12^2 = 256 + 144 = 400 = 20^2 = BC^2$.
Therefore, by the Pythagorean Theorem, $\triangle ABC$ is right-angled and $\angle A = 90°$.

We can use AB as the base and AC as the height to determine the area of $\triangle ABC$.

$$\text{area } \triangle ABC = (AB)(AC) \div 2 = (16)(12) \div 2 = 96 \text{ cm}^2$$

In the circle, BC is a diameter and $BC = 20$. Therefore, the radius of the circle is 10 cm.

We can now determine the area of the circle.

$$\text{area of circle} = \pi r^2 = \pi(10)^2 = 100\pi$$

The unshaded area is calculated by subtracting the area of the triangle from the area of the circle.

$$\text{Unshaded area} = (100\pi - 96) \text{ cm}^2 \doteq 218.16 \text{ cm}^2$$

$\therefore$The unshaded area is $(100\pi$ - 96) cm^2 or approximately 218.16 cm^2.