



Problem of the Week

Grade 9 and 10

That is Sum Probability Solution

Problem

For any three digit integer greater than or equal to 100, the sum of the digits is calculated. Determine the probability that the sum of the digits is 10.

Solution

First we should determine the number of three digit integers greater than or equal to 100. The number 999 is the largest three digit number so from 1 to 999 there are 999 numbers. This includes 99 numbers from 1 to 99 that must be excluded. So there are $999 - 99 = 900$ three digit numbers.

We now must determine how many of these numbers have a digit sum of 10.

If the hundred's digit is 1, the remaining digits must sum to 9. It is then easy to list the possibilities, {109, 118, 127, 136, 145, 154, 163, 172, 181, 190}, 10 numbers in total.

If the hundred's digit is 2, the remaining digits must sum to 8. It is then easy to list the possibilities, {208, 217, 226, 235, 244, 253, 262, 271, 280}, 9 numbers in total.

We can proceed in this manner and show that there are 8 numbers with a hundred's digit of 3, 7 numbers with a hundred's digit of 4, 6 numbers with a hundred's digit of 5, 5 numbers with a hundred's digit of 6, 4 numbers with a hundred's digit of 7, 3 numbers with a hundred's digit of 8, and 2 numbers with a hundred's digit of 9.

There are a total of $10 + 9 + 8 + \dots + 4 + 3 + 2 = 54$ three digit numbers with a digit sum of 10. (We can calculate the sum mentally or using a calculator. Or we could use the result that the sum of the natural numbers from 1 to n can be calculated using the formula $\frac{(n)(n+1)}{2}$. We could find the sum of the natural numbers from 1 to 10 and subtract 1 from the result since the number 1 is missing from the above sum. So the sum is $\frac{(10)(11)}{2} - 1 = \frac{110}{2} - 1 = 55 - 1 = 54$.)

The probability of a digit sum of 10 is $\frac{\text{number of numbers with digit sum 10}}{\text{number of three digit numbers}} = \frac{54}{900} = \frac{3}{50}$.

\therefore the probability that 10 is the sum of the digits of a three digit number is $\frac{3}{50}$.

