



Problem of the Week

Grade 9 and 10

The Phantom Menace: Students Quitting

Solution

Problem

A large group of students came out to an information meeting for the upcoming school musical, Phantom of the Math Room. Early in the meeting 15 girls left. Twice as many boys as girls remained. Later, after the 15 girls left, $\frac{3}{4}$ of the boys and $\frac{1}{3}$ of the remaining girls left. This left 14 more girls than boys. All of the remaining students stuck it out and went on to perform in an amazing production. How many students remained to perform in the school musical, Phantom of the Math Room?

Solution

Let b represent the number of boys and g represent the number of girls at the start of the information meeting.

Since 15 girls leave, $g - 15$ girls remain. The number of boys is twice the number of girls at this point so $b = 2(g - 15)$ which simplifies to $b = 2g - 30$. (1)

Then $\frac{3}{4}$ of the boys depart leaving $\frac{1}{4}$ of the boys or $\frac{1}{4}b$. $\frac{1}{3}$ of the girls remaining depart leaving $\frac{2}{3}$ of the girls remaining or $\frac{2}{3}(g - 15)$. Now the number of girls is 14 more than the number of boys so $\frac{2}{3}(g - 15) = \frac{1}{4}b + 14$. Multiplying by 12, the equation simplifies to $8(g - 15) = 3b + 168$. This further simplifies to $8g - 120 = 3b + 168$. (2)

At this point we can use substitution or elimination to solve the system of equations. In this case, we substitute (1) into (2) for b to solve for g .

$$\begin{aligned}8g - 120 &= 3(2g - 30) + 168 \\8g - 120 &= 6g - 90 + 168 \\2g &= 198 \\g &= 99\end{aligned}$$

Substituting $g = 99$ into (1), $b = 2(99) - 30 = 198 - 30 = 168$.

The original number of students is $b + g = 168 + 99 = 267$.

The number of boys remaining is $\frac{1}{4}b = \frac{1}{4}(168) = 42$.

The number of girls remaining is $\frac{2}{3}(g - 15) = \frac{2}{3}(99 - 15) = \frac{2}{3}(84) = 56$.

The total number of students remaining to successfully perform in the musical is $42 + 56 = 98$.

The show was a success even though only 98 of the initial 267 students remained.

