Problem of the Week
Grade 7 and 8

This is Some Sum
Solution

Problem

In the late 1700s, Gauss was asked to find the sum of the numbers from 1 to 100. Gauss quickly gave the answer 5 050. He did this by looking at patterns. Instead of finding the sum of the numbers 1 to 100, can you find the sum of the **digits** of the numbers from 1 to 100?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Solution

(1) Each of the ten columns has a units digit that occurs ten times.

$$\begin{aligned}\text{Sum of ALL units digits} &= 10(1)+10(2)+10(3)+\cdots+10(9)+10(0)\\ &= 10(1+2+3+\cdots+9+0)\\ &= 10(45)\\ &= 450\end{aligned}$$

(2) Each of the ten columns has a ten's digit from 0 to 9.

$$\begin{aligned}\text{Sum of ALL tens digits} &= 10(0+1+2+3+\cdots+8+9)\\ &= 10(45)\\ &= 450\end{aligned}$$

(3) The number 100 is the only number with a hundreds digit. We need to add 1 to our final sum.

(4) Now we add our results from (1), (2), and (3) to obtain the required sum.

$$\begin{aligned}\text{Sum of digits} &= \text{Units digit sum} + \text{Tens digit sum} + \text{Hundreds Digit}\\ &= 450+450+1\\ &= 901\end{aligned}$$

Therefore the sum of the digits of the numbers from 1 to 100 is 901.