



# Problem of the Week

## Grade 11 and 12

### The Quest For Ann Sirs?

### Solution

#### Problem

Ann Sirs has already written three mathematics tests, worth 100 marks each. The average of her marks on her first two tests is calculated and then averaged with her third test mark giving an average of 86%. The average of her marks on her second and third tests is calculated and then averaged with her first test mark giving an average of 79.5%. The average of her marks on her first and third tests is calculated and then averaged with her second test mark giving an average of 81.5%. Her fourth mathematics test is tomorrow and it is also out of 100. What mark does Ann need on her test to achieve an overall average of 85% in her mathematics course?

#### Solution

Let  $a$  represent Ann's first mark,  $b$  represent Ann's second mark,  $c$  represent Ann's third mark and  $d$  represent Ann's fourth mark.

When the average of her first and second marks is averaged with her third test mark, the new average is 86 so  $\frac{\frac{a+b}{2}+c}{2} = 86$ . Multiplying by 2 gives  $\frac{a+b}{2} + c = 172$ . Multiplying by 2 again yields  $a + b + 2c = 344$ . (1)

When the average of her second and third marks is averaged with her first test mark, the new average is 79.5 so  $\frac{\frac{b+c}{2}+a}{2} = 79.5$ . Multiplying by 2 gives  $\frac{b+c}{2} + a = 159$ . Multiplying by 2 again yields  $b + c + 2a = 318$ . (2)

When the average of her first and third marks is averaged with her second test mark, the new average is 81.5 so  $\frac{\frac{a+c}{2}+b}{2} = 81.5$ . Multiplying by 2 gives  $\frac{a+c}{2} + b = 163$ . Multiplying by 2 again yields  $a + c + 2b = 326$ . (3)

By adding equations (1), (2) and (3) we obtain  $4a + 4b + 4c = 988$  which simplifies to  $a + b + c = 247$  after dividing by 4. This means that the sum of her first three marks is 247.

To obtain an average of 85% Ann needs  $\frac{a+b+c+d}{4} = 85$ . Substituting  $a + b + c = 247$  we obtain  $\frac{247+d}{4} = 85$ . Multiplying by 4 gives  $247 + d = 340$  or  $d = 93$ .

**$\therefore$  to obtain an average of 85% Ann needs 93 on tomorrow's test.**

It would be a straight forward process to determine Ann's first three test marks but our quest did not ask us to do this. However, for the curious, on her first test Ann got 71, on her second test Ann got 79, and on her third test Ann got 97.

