Problem of the Week
Grade 11 and 12

POWERful
Solution

Problem

When $n = 6^{2011}$ is expressed as an integer, what are its last two digits?

Solution

Start by examining the last two digits of various powers of 6.

$$\begin{aligned}
6^1 &= \mathbf{06}\\
6^2 &= \mathbf{36}\\
6^3 &= 2\mathbf{16}\\
6^4 &= 1\ 2\mathbf{96}\\
6^5 &= 7\ 7\mathbf{76}\\
6^6 &= 46\ 6\mathbf{56}\\
6^7 &= 279\ 9\mathbf{36}\\
6^8 &= 1\ 679\ 6\mathbf{16}\\
6^9 &= 10\ 077\ 6\mathbf{96}\\
6^{10} &= 60\ 466\ 1\mathbf{76}\\
6^{11} &= 362\ 797\ 0\mathbf{56}
\end{aligned}$$

Notice that the last two digits repeat every five powers of 6 starting with the 2^{nd} power of 6. The pattern continues. 6^{12} ends with 36, 6^{13} ends with 16, 6^{14} ends with 96, 6^{15} ends with 76, 6^{16} ends with 56, and so on. Starting with the second power of 6, every five consecutive powers of 6 will have the last two digits 36, 16, 96, 76, and 56.

We need to determine the number of complete cycles in 2011 by first subtracting 1 to allow for 06 at the beginning of the list and then dividing $2011 - 1$ or 2010 by 5.

$$\frac{2010}{5} = 402$$

So there are 402 complete cycles through the digit pattern to get to 6^{2011}. Since 6^{2011} is the last number in the pattern it ends in 56.

Therefore 6^{2011} ends with the digits 56.