



Problem of the Week

Grade 11 and 12

How Many Children Are There Anyway?

Solution

Problem

A man died leaving some money in his estate. All of this money was to be divided among his children in the following manner: $\$x$ to the first born plus $\frac{1}{16}$ of what remains, $\$2x$ to the second born plus $\frac{1}{16}$ of what then remains, $\$3x$ to the third born plus $\frac{1}{16}$ of what then remains, and so on. When the distribution of the money was complete, each child received the same amount and no money was left over. Determine the number of children.

Solution

Let T be the total value of the estate. $T > 0$ since there is money in the estate.

Let y be the amount of money given to each child. $y > 0$ since each child receives money.

Then $\frac{T}{y}$ is the number of children.

The first child receives x to begin with. There would be $(T - x)$ left at this point. The first child then receives $\frac{1}{16}$ of the remaining amount $(T - x)$. Therefore, the first child receives $y = x + \frac{1}{16}(T - x)$.

The second child receives $2x$ to begin with. There would now be $(T - y - 2x)$ left in the estate. This represents the original amount minus the first child's full share minus the amount received so far by the second child. The second child then receives $\frac{1}{16}$ of the remaining amount $(T - y - 2x)$. Therefore, the second child receives $y = 2x + \frac{1}{16}(T - y - 2x)$.

But each child receives the same amount.

$$\begin{aligned}\therefore x + \frac{1}{16}(T - x) &= 2x + \frac{1}{16}(T - y - 2x) \\ x + \frac{1}{16}T - \frac{1}{16}x &= 2x + \frac{1}{16}T - \frac{1}{16}y - \frac{1}{16}(2x) \\ x - \frac{1}{16}x &= 2x - \frac{1}{16}y - \frac{2}{16}x\end{aligned}$$

Multiply each side of the equation by 16.

$$16x - x = 32x - y - 2x$$

$$15x = 30x - y$$

$$y = 15x$$

Therefore each child receives $\$15x$.

(The solution continues on the next page.)





Substituting $15x$ for y into the equation $y = x + \frac{1}{16}(T - x)$ we obtain

$$x + \frac{1}{16}(T - x) = 15x$$

$$\frac{1}{16}(T - x) = 14x$$

$$T - x = 224x$$

$$T = 225x$$

Therefore the total value of the estate is $225x$.

We can now determine the number of children $\frac{T}{y} = \frac{225x}{15x} = 15$.

Therefore the father had 15 children.

Notice, we did not need to know T , the total value of the estate. It turns out that once we know x , we can determine each child's share, $15x$, and the total value of the estate, $T = 225x$.

