

Problem of the Week Problem C and Solution Arranging Tiles 1

Problem

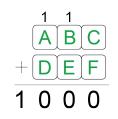
Ana has nine tiles, each with a different integer from 1 to 9 on it. Ana creates larger numbers by placing tiles side by side. For example, using the tiles 3 and 7, Ana can create the 2-digit number 37 or 73. Using six of her tiles, Ana forms two 3-digit numbers that add to 1000. What is the largest possible 3-digit number that she could have used?

Solution

We will use the letters A, B, C, D, E, and F to represent the integers on the six chosen tiles, letting the two 3-digit numbers be ABC and DEF. Then we will determine the largest possible 3-digit number ABC.

Looking at the ones column, since C and F are both digits from 1 to 9 and add to a number that ends in 0, their sum must be 10. (Their sum cannot be zero since neither C nor F is zero, and their sum cannot be 20 or more since C and F are each less than 10.) Thus, C + F = 10. Therefore, there is a carry of 1 into the tens column. Similarly, the sum in the tens column must also be 10, so B + E + 1 = 10, or B + E = 9. Therefore, there is a carry of 1 into the tens of 1 into the hundreds column. Thus, A + D + 1 = 10, or A + D = 9.

 $\frac{ABC}{+DEF}$



To determine the largest possible 3-digit number ABC, A must be as large as possible. We have the following tiles: 1, 2, 3, 4, 5, 6, 7, 8, and 9. Since A + D = 9, A is largest when A = 8 and D = 1.

The next step is to make B as large as possible. We are left with the following tiles: 2, 3, 4, 5, 6, 7, and 9. Since B + E = 9, B is largest when B = 7 and E = 2.

Finally, we need to make C as large as possible. We are left with the following tiles: 3, 4, 5, 6, and 9. Since C + F = 10, then C is largest when C = 6 and F = 4.

Therefore, the largest possible 3-digit number ABC is 876.

Indeed, we can check that when ABC is 876, we have DEF equal to 124, and ABC + DEF = 876 + 124 = 1000.