Problem of the Week Problem B and Solution Don't Get Mowed Over!

Problem

Jon has a grass cutting business. Three of the lawns that he cuts are shown. The lawns are shaded in green and all angles are right angles. His lawnmower can cut a swath of width 1 metre.



- (a) Which lawn will take the longest to cut? Explain your reasoning.
- (b) His lawnmower travels at 3 km per hour. What area of lawn, in m², can he cut in one hour?
- (c) How long, in minutes, will it take him to cut each lawn?

Solution

(a) Since each lawn is composed of regions that have integer lengths, in metres, and since the lawnmower can cut a swath of width 1 metre, we can compare the areas of the lawns to determine which will take the longest to cut. The Ngans' lawn is a rectangle measuring 25 m × 40 m. Its total area is therefore 25 m × 40 m = 1000 m^2 .

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The Jones' lawn can be divided into two smaller rectangles as shown.



The total area of the Jones' lawn is equal to the sum of the area of the rectangle on the left and the area of the rectangle on the right. The rectangle on the left has area equal to $40 \text{ m} \times 20 \text{ m} = 800 \text{ m}^2$. The rectangle on the right has area equal to $20 \text{ m} \times 25 \text{ m} = 500 \text{ m}^2$. Thus, the total area of the Jones' lawn is equal to $800 \text{ m}^2 + 500 \text{ m}^2 = 1300 \text{ m}^2$.

The area of the Bhutus' lawn can be found by finding the area of the outer rectangle and subtracting the area of the inner rectangle. The area of the outer rectangle is equal to $50 \text{ m} \times 35 \text{ m} = 1750 \text{ m}^2$. The area of the inner rectangle is equal to $10 \text{ m} \times 25 \text{ m} = 250 \text{ m}^2$. Thus, the total area of the Bhutus' lawn is equal to $1750 \text{ m}^2 - 250 \text{ m}^2 = 1500 \text{ m}^2$.

Since the Bhutus' lawn has the largest area, it will take the longest to cut.

NOTE: To determine the area of the Bhutus' lawn, we could have alternatively divided the lawn into smaller rectangles, and summed the areas of those rectangles.

- (b) Jon's mower is 1 m wide and it travels at 3 km/h, or 3000 m/h. Therefore, he can cut 1 m \times 3000 m = 3000 m² in one hour.
- (c) The Ngans' lawn has area equal to 1000 m². Thus, it would take $1000 \div 3000 = 0.333$ (or $\frac{1}{3}$) of an hour, which is $\frac{1}{3} \times 60 = 20$ minutes to cut the lawn.

The Jones' lawn has area equal to 1300 m^2 . It would take $1300 \div 3000 = 0.4333 \text{ (or } \frac{13}{30}\text{)}$ of an hour, which is $\frac{13}{30} \times 60 = 26$ minutes to cut the lawn.

The Bhutus' lawn has area equal to 1500 m^2 . It would take $1500 \div 3000 = 0.5 \text{ (or } \frac{1}{2}\text{)}$ of an hour, which is $\frac{1}{2} \times 60 = 30$ minutes to cut the lawn.