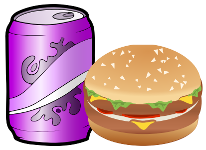
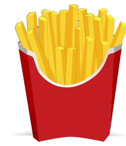


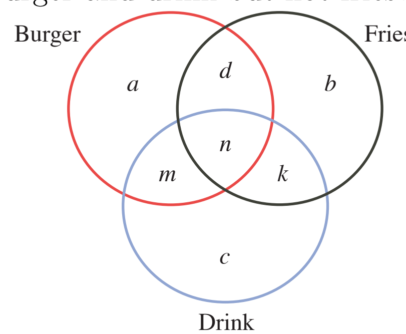
Problem of the Week
Problem E and Solution
Food Truck Fare



Problem

Food To Go sells only burgers, fries and drinks from their food truck. Today, exactly 120 people made a purchase. Half of the people purchased at least a burger, $\frac{1}{4}$ purchased at least fries but $\frac{1}{3}$ of the customers purchased only a drink. Of the people who bought a burger, $\frac{4}{5}$ of them bought at least one other item. How many people purchased a burger and drink but not fries?

Solution



This problem can be solved nicely using a Venn diagram (shown to the right), and we encourage those of you familiar with Venn diagrams to do so. In our solution we will set up various equations and solve for the required variable.

Let a be the number of people who purchased a burger only.
Let b be the number of people who purchased fries only.
Let c be the number of people who purchased a drink only.

Let d be the number of people who purchased a burger and fries but not a drink.
Let k be the number of people who purchased fries and a drink but not a burger.
Let m be the number of people who purchased a burger and a drink but not fries.
Let n be the number of people who purchased a burger, fries and a drink.
We need to determine m.

We know that 120 people made a purchase, so $a + b + c + d + k + m + n = 120$ (1).
We are given that half of the people purchased a burger, so $\frac{1}{2} \times 120 = 60$ people purchased a burger. These people may have also purchased fries or a drink or both or neither. This tells us that $a + d + m + n = 60$ (2).

We are given that $\frac{1}{4}$ of the people purchased fries, so $\frac{1}{4} \times 120 = 30$ people purchased fries. These people may have also purchased a burger or a drink or both or neither. This tells us that $b + d + k + n = 30$ (3).

We are given that $\frac{1}{3}$ of the people purchased only a drink, so $\frac{1}{3} \times 120 = 40$ people purchased only a drink. Therefore, $c = 40$.

We are given that, of the people who bought a burger, $\frac{4}{5}$ bought at least one other item. So, $\frac{4}{5}$ of the 60 people who purchased a burger bought at least one other item. In other words, $\frac{4}{5} \times 60 = 48$ also bought fries, a drink or both. Therefore, $d + m + n = 48$ (4).

Subtracting (4) from (2), we see that $a = 12$.

Substituting $a = 12$ and $c = 40$ into (1), we have $12 + b + 40 + d + k + m + n = 120$, and thus $b + d + k + m + n = 68$, or $(b + d + k + n) + m = 68$. From (3) we know $b + d + k + n = 30$, and so $30 + m = 68$, thus $m = 38$. Therefore, 38 people purchased a burger and drink but not fries.

We have determined what was required and can stop here. We do not need to solve for the remaining variables. It actually turns out that in this problem there is not enough information given to determine the values of all of the remaining variables.