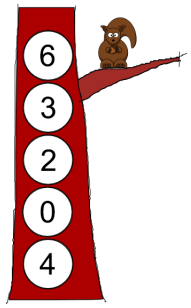
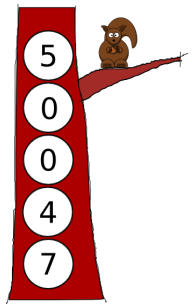
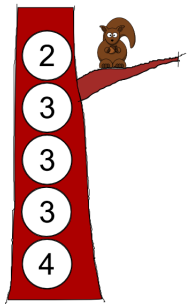
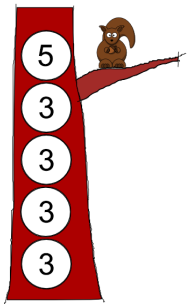
Problem of the Week
Problem E and Solution
Squirrel!

Problem

Some squirrels live in a tree with five big holes located one above the other. Every day, in the morning, each squirrel counts the number of **other** squirrels in their present hole and how many squirrels are in the adjacent holes above and below their present hole. Every night, each squirrel secretly either stays where they are or moves to a hole above or below their current hole. A squirrel in the lowest hole can only choose to stay in the lowest hole or move up one hole. A squirrel in the highest hole can only choose to stay in the highest hole or move down one hole. The squirrels move independently and make their moves according to the following rules:

1. If the number of **other** squirrels in their present hole is less than the **total** number of squirrels in either the hole above or the hole below them, then the squirrel stays where it is.
2. If the number of **other** squirrels in their present hole is the same as the **total** number of squirrels in the hole above them, then the squirrel moves up. If the number of **other** squirrels in their present hole is the same as the **total** number of squirrels in the hole below them, then the squirrel stays where it is. Squirrels always prefer to live higher in trees.
3. If the number of **other** squirrels in their present hole is more than the **total** number of squirrels in either the hole above or the hole below them, then the squirrel moves to the hole which gives them the fewest number of **other** squirrels than their present hole. In the event that there are the same number of squirrels in each choice, remember that squirrels prefer to live higher in trees.

Each day these rules are applied until all of the squirrels are in the same hole. In some instances this will never happen! For each of the following scenarios, determine the number of days from the initial day that it takes until all of the squirrels are in the same hole. The number in each circle represents the **total** number of squirrels in each hole on the initial day. The solution to the example scenario was provided with the problem.

Example Scenario	Scenario 1	Scenario 2	Scenario 3
			

Solution

For Scenario 1, all 16 squirrels will be in the second hole from the top 5 days after the initial day.

For Scenario 2, the squirrels will never end up in the same hole. On day 3 there will be 8 squirrels in the top hole and 7 squirrels in the fourth hole from the top. On day 4 there will be 8 squirrels in the second hole from the top and 7 squirrels in the third hole from the top. On day 5 the squirrels move back into the holes they were in on day 3 and on day 6 the squirrels move back into the holes they were in on day 4. The cycle repeats.

For Scenario 3, all 17 squirrels will be in the second hole from the top 7 days after the initial day.

Complete explanations for the above answers are contained on the following page.

We will use ordered quintuples to represent the number of squirrels in each hole at the start of any day. For the ordered quintuple (a, b, c, d, e), a is the number of squirrels in the top hole, b is the number of squirrels in the second hole from the top, c is the number of squirrels in the third hole from the top, d is the number of squirrels in the fourth hole from the top, and e is the number of squirrels in the bottom hole.

Solution to Scenario 1

Initially, we have $(5, 0, 0, 4, 7)$. The 5 squirrels in the top hole move to the second hole from the top (0 others is better than 4 others). The 4 squirrels in the fourth hole from the top move to the third hole from the top (0 others is better than 3 others or 7 others). The 7 squirrels in the bottom hole move up to the fourth hole from the top (4 others is better than 6 others). So at the start of Day 1 we have $(0, 5, 4, 7, 0)$.

The 5 squirrels in the second hole from the top move up to the top hole (0 others is better than 4 others or 4 others). The 4 squirrels in the third hole from the top stay where they are (3 others is better than 7 others or 5 others). The 7 squirrels in the fourth hole from the top move to the bottom hole (0 others is better than 4 others or 6 others). So at the start of Day 2 we have $(5, 0, 4, 0, 7)$.

The 5 squirrels in the top hole move to the second hole from the top (0 others is better than 4 others). The 4 squirrels in the third hole from the top move to the second hole from the top (0 others higher is better than 3 others or 0 others lower). The 7 squirrels in the bottom hole move up to the fourth hole from the top (0 others is better than 6 others). So at the start of Day 3 we have $(0, 9, 0, 7, 0)$.

The 9 squirrels in the second hole from the top move up to the top hole (0 others higher is better than 8 others or 0 others lower). The 7 squirrels in the fourth hole from the top move to the third hole from the top (0 others higher is better than 6 others or 0 others lower). So at the start of Day 4 we have $(9, 0, 7, 0, 0)$.

The 9 squirrels in the top hole move to the second hole from the top (0 others is better than 8 others). The 7 squirrels in the third hole from the top move to the second hole from the top (0 others higher is better than 6 others or 0 others lower.) So at the start of Day 5 we have $(0, 16, 0, 0, 0)$. All the squirrels are now in the same hole.

Solution to Scenario 2

Initially, we have $(2, 3, 3, 3, 4)$. The 2 squirrels in the top hole do not move (1 other is better than 3 others). The 3 squirrels in the second hole from the top will move up to the top hole (2 others higher is better than 2 others or 3 others). The 3 squirrels in the third hole from the top stay where they are (2 others is better than 3 others or 3 others). The 3 squirrels in the fourth hole from the top stay where they are (2 others is better than 3 others or 4 others). The 4 squirrels in the bottom hole move to the fourth hole from the bottom (3 others higher is better than 3 others). So at the start of Day 1 we have $(5, 0, 3, 7, 0)$.

The 5 squirrels in the top hole will move to the second hole from the top (0 others are better than 4 others). The 3 squirrels in the third hole from the top will move to the second hole from the top (0 others is better than 2 others or 7 others). The 7 squirrels in the fourth hole from the top will move to the bottom hole (0 others is better than 3 others or 6 others). So at the start of Day 2 we have $(0, 8, 0, 0, 7)$.

The 8 squirrels in the second hole from the top will move up to the top hole (0 others higher is better than 7 others or 0 others lower). The 7 squirrels in the bottom hole will move to the fourth hole from the top (0 others are better than 6 others). So at the start of Day 3 we have $(8, 0, 0, 7, 0)$.

The 8 squirrels in the top hole will move to the second hole from the top (0 others is better than 7 others). The 7 squirrels in the fourth hole from the top will move to the third hole from the top.

(0 others higher is better than 6 others or 0 others lower). So at the start of Day 4 we have $(0, 8, 7, 0, 0)$.

The 8 squirrels in the second hole from the top will move up to the top hole (0 others higher is better than 7 others or 7 others lower). The 7 squirrels in the third hole from the top will move to the fourth hole from the top (0 others are better than 8 others or 6 others). So at the start of Day 5 we have $(8, 0, 0, 7, 0)$.

This is the same as the start of Day 3 and so Day 6 will start the same as Day 4. We will now repeat these days and never get all the squirrels in the same hole.

Solution to Scenario 3

Initially, we have $(5, 3, 3, 3, 3)$. The 5 squirrels in the top hole move to the second hole from the top (3 others is better than 4 others). The 3 squirrels in the second hole from the top stay where they are (2 others is better than 5 others or 3 others). The 3 squirrels in the third hole from the top stay where they are (2 others is better than 3 others or 3 others). The 3 squirrels in the fourth hole from the top stay where they are (2 others is better than 3 others or 3 others). The 3 squirrels in the bottom hole stay where they are (2 others is better than 3 others). So at the start of Day 1 we have $(0, 8, 3, 3, 3)$.

The 8 squirrels in the second hole from the top will move up to the top hole (0 others up is better than 7 others or 3 others). The 3 squirrels in the third hole from the top stay where they are (2 others is better than 8 others or 3 others). The 3 squirrels in the fourth hole from the top stay where they are (2 others is better than 3 others or 3 others). The 3 squirrels in the bottom hole stay where they are (2 others is better than 3 others). So at the start of Day 2 we have $(8, 0, 3, 3, 3)$.

The 8 squirrels in the top hole will move to the second hole from the top (0 others is better than 7 others). The 3 squirrels in the third hole from the top will move to the second hole from the top (0 others is better than 2 others or 3 others). The 3 squirrels in the fourth hole from the top stay where they are (2 others is better than 3 others or 3 others). The 3 squirrels in the bottom hole stay where they are (2 others is better than 3 others). So at the start of Day 3 we have $(0, 11, 0, 3, 3)$.

The 11 squirrels in the second hole from the top will move to the top hole (0 others higher is better than 10 others or 0 others lower). The 3 squirrels in the fourth hole from the top will move to the third hole from the top (0 others is better than 2 others or 3 others). The 3 squirrels in the bottom hole stay where they are (2 others is better than 3 others). So at the start of Day 4 we have $(11, 0, 3, 0, 3)$.

The 11 squirrels in the top hole will move to the second hole from the top (0 others is better than 10 others). The 3 squirrels in the third hole from the top will move to the second hole from the top (0 others higher is better than 2 others or 0 others lower). The 3 squirrels in the bottom hole will move the fourth hole from the top (0 others is better than 2 others). So at the start of Day 5 we have $(0, 14, 0, 3, 0)$.

The 14 squirrels in the second hole from the top will move to the top hole (0 others higher is better than 13 others or 0 others lower). The 3 squirrels in the fourth hole from the top will move to the third hole from the top (0 others higher is better than 2 others or 0 others lower). So at the start of Day 6 we have $(14, 0, 3, 0, 0)$.

The 14 squirrels in the top hole will move to the second hole from the top (0 others is better than 13 others). The 3 squirrels in the third hole from the top will move to the second hole from the top (0 others higher is better than 2 others or 0 others down). So at the start of Day 7 we have $(0, 17, 0, 0, 0)$. All the squirrels are now in the same hole.

See the next page for further discussion.

For Further Thought

1. In the example scenario and scenarios 1 and 3 the squirrels all came together in the second hole from the top. If the squirrels do come together in the same hole, is it possible for this convergence to happen in any other hole then the second hole from the top?

2. In scenario 3, the squirrels were in the same hole for the first time at the start of Day 7. Is this the largest number of days it can take to converge?