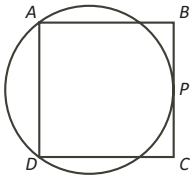
Problem of the Week
Problem E and Solution
Off Centre

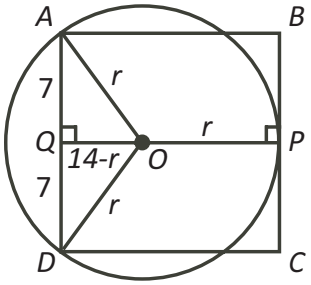


Problem

Square $ABCD$ has sides of length 14 cm. The square is not centered on the circle. It is offset in such a way that A and D are on the circle and side BC is tangent to the circle at point P. Determine the radius of the circle.

Solution

The diagram to the right will be justified in this solution.
Let O be the centre of the circle and r be the radius.



Construct line segment PQ perpendicular to CB with P on side BC and Q on side AD of the square. Since CB is tangent to the circle, PQ must pass through the centre of the circle, O. Therefore, $PO = r$.

Since $PQ \perp BC$, $PQ \parallel AB$ and $PQ = AB = 14$, then $QO = PQ - PO = 14 - r$.
Since A and D are on the circle, $AO = DO = r$.

Using the Pythagorean Theorem, $AQ^2 = AO^2 - QO^2 = r^2 - (14 - r)^2$ and $DQ^2 = DO^2 - QO^2 = r^2 - (14 - r)^2$. Therefore $AQ^2 = DQ^2$ and $AQ = DQ$ follows. Since $AQ = DQ$ and $AQ + QD = AD = 14$, we can substitute to obtain $AQ + AQ = 2AQ = 14$ or $AQ = 7$.

Using the Pythagorean Theorem in $\triangle AQO$,

$$\begin{aligned} AO^2 &= AQ^2 + QO^2 \\ r^2 &= 7^2 + (14 - r)^2 \\ r^2 &= 49 + 196 - 28r + r^2 \\ 28r &= 245 \\ r &= \frac{245}{28} \\ r &= \frac{35}{4} \\ r &= 8.75 \text{ cm} \end{aligned}$$

The radius of the circle is $\frac{35}{4}$ cm or 8.75 cm.