



Problem of the Week

Problem D and Solution

Leaps and Bounds

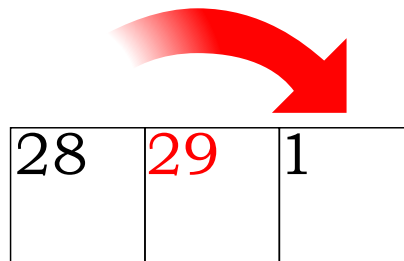
Problem

Most people believe a year is equivalent to 365 days. In actuality, it is slightly more than 365 days. To account for this extra time, we use leap years, which are years containing one extra day. The following rule is used to determine if a year is a leap year:

A year is a leap year if it is

- divisible by four, and
- not divisible by 100, unless it is also divisible by 400.

A year greater than 2000 is chosen at random, what is the probability that it is a leap year?



Solution

The probability of an event occurring is calculated as the number of favourable outcomes divided by the total number of possible outcomes. This is an issue in our problem because the number of years greater than 2000 is infinite. However, in this case, the cycle of leap years repeats every 400 years. For example, since 2044 is a leap year so is 2444.

The number of leap years in a cycle can be counted this way.

The multiples of 4 in a 400 year cycle is $\frac{400}{4} = 100$. However, we have counted the multiples of 100 so we need to subtract these multiples which is $\frac{400}{100} = 4$ to get $100 - 4 = 96$. We now need to add back the the multiples of 400 which is $\frac{400}{400} = 1$ to get $96 + 1 = 97$.

Therefore for every 400 year cycle, 97 of these years will be a leap year and the probability of getting a leap year is $\frac{97}{400} = 0.2425$.

