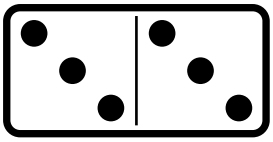

Problem of the Week
Problem D and Solution
Something's Missing

Problem

Dominic and Dmitri divide a 6-set of dominoes into two unequal piles. They then realize that one domino is missing from the set. Dimitri counts all of the pips on the dominoes in the first pile. He counts that there are a total of 91 pips. Dominic counts all of the pips on the dominoes in the second pile. He counts that there are a total of 67 pips. Dominic also notes that all the double dominoes are accounted for. Which domino is missing from the set?

Solution

We first determine which dominoes are in a 6-set and calculate the total number of pips on all of the dominoes in the set. In a 6-set of dominoes, the number of pips on each end of a domino tile can range from 0 to 6. Since rotating a domino tile does not change the domino, let's orient each tile so that the smaller number on non double dominoes is always on the left end of the tile.

For each possible number on the left side of the domino, let's examine the possible numbers that can occur on the right side and compile this information in a table.

Number on Left	Possible Numbers on Right	Sum of Pips on Dominoes
0	0, 1, 2, 3, 4, 5, 6	$1+2+3+4+5+6=21$
1	1, 2, 3, 4, 5, 6	$6(1)+1+2+3+4+5+6=27$
2	2, 3, 4, 5, 6	$5(2)+2+3+4+5+6=30$
3	3, 4, 5, 6	$4(3)+3+4+5+6=30$
4	4, 5, 6	$3(4)+4+5+6=27$
5	5, 6	$2(5)+5+6=21$
6	6	$1(6)+6=12$

Therefore, the total number of pips on the dominoes in a 6-set is

$$21+27+30+30+27+21+12=168.$$

Since the total number of pips in the first pile is 91, and the total number of pips in the second pile is 67, the sum is 158. That leaves a total of 10 pips on the missing tile.

In a 6-set of dominoes, the only tiles with 10 pips are $[5,5]$ and $[4,6]$. Since all the double dominoes are present, then the $[5,5]$ is present. Therefore, the missing tile must be the $[4,6]$ tile.