Problem of the Week
Problem D and Solution
A Different Difference

Problem
The non-negative difference between two numbers \( a \) and \( b \) is \( b - a \) or \( a - b \), whichever is greater than or equal to zero. For example, the non-negative difference between 24 and 64 is 40. In a sequence that begins 74, 60, 14, 46, 32, \( \cdots \), each number after the second number is obtained by finding the non-negative difference between the previous 2 numbers. Determine the sum of the first 1300 numbers in the sequence.

Solution
We will start by generating more terms of the sequence in an attempt to find a pattern.

Using the rule for creating the sequence, we obtain

\[
74, 60, 14, 46, 32, 14, 18, 4, 14, 10, 4, 6, 2, 4, 2, 2, 0, 2, 2, 0, 2, 2, 0, \cdots
\]

The first 14 terms of the sequence have no apparent pattern. The values of the 15\(^{th}\), 16\(^{th}\) and 17\(^{th}\) terms repeat as the sequence is extended. So the 15\(^{th}\) term, 18\(^{th}\) term, 21\(^{st}\) term, and so on, all equal the first 2 in the string 2,2,0.

The first 14 terms of the sequence are followed by \( n \) groups of 2,2,0. What is the value of \( n \)? We want a total of 1300 terms. If we remove the first 14 terms, we require \( 1300 - 14 = 1286 \) more terms. If we divide 1286 by 3 we are able to determine how many complete strings of 2,2,0 are needed. The result is \( 1286 \div 3 = 428\frac{2}{3} \). This tells us that we need 428 complete copies of 2,2,0 and \( \frac{2}{3} \) of a copy of 2,2,0, namely 2,2.

The required sum is the sum of the first 14 terms plus \( 428 \times (2 + 2 + 0) + 2 + 2 \). The sum of the first 14 terms is 302. The sum of the first 1300 terms of the sequence is \( 302 + 428 \times 4 + 4 = 2018 \).

The sum of the first 1300 terms of the sequence is 2018.