Can You Repeat That A Little Later?

**Problem**

When $\frac{1}{70 \, 000 \, 000}$ is written as a decimal, what digit occurs in the $2018^{\text{th}}$ place after the decimal point?

**Solution**

Notice that $\frac{1}{70 \, 000 \, 000} = \frac{1}{10 \, 000 \, 000} \times \frac{1}{7} = 0.000 \, 000 \, 1 \times \frac{1}{7}$.

Also, note that $\frac{1}{7} = 0.\overline{142857}$. That is, when $\frac{1}{7}$ is written as a decimal, the digits after the decimal point occur in repeating blocks of the 6 digits 142857.

Therefore, $\frac{1}{70 \, 000 \, 000} = 0.000 \, 000 \, 1 \times \frac{1}{7} = 0.000 \, 000 \, 1 \times 0.\overline{142857} = 0.000 \, 000 \, 01\overline{42857}$.

That is, when $\frac{1}{70 \, 000 \, 000}$ is written as a decimal, the digits after the decimal point will be seven 0’s followed by repeating blocks of the six digits 142857.

We see the decimal representation of $\frac{1}{70 \, 000 \, 000}$ has the same repetition as that for $\frac{1}{7}$, but the pattern is shifted over 7 places. Therefore, the $2018^{\text{th}}$ digit after the decimal point when $\frac{1}{70 \, 000 \, 000}$ is written as a decimal is the same as the $(2018 - 7) = 2011^{\text{th}}$ digit after the decimal point when $\frac{1}{7}$ is written as a decimal.

Since $\frac{2011}{6} = 335\frac{1}{6}$, then the $2011^{\text{th}}$ digit after the decimal point occurs after 335 blocks of the repeating digits have been used. In 335 blocks of six digits, there are $335 \times 6 = 2010$ digits in total.

Therefore, the $2011^{\text{th}}$ digit is one digit into the $336^{\text{th}}$ block of repeating digits, so it must be a 1.

The $2018^{\text{th}}$ digit after the decimal point in the decimal representation of $\frac{1}{70 \, 000 \, 000}$ is the same as the $2011^{\text{th}}$ digit after the decimal point in the decimal representation of $\frac{1}{7}$ and is therefore a 1.