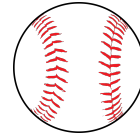




Problem of the Week

Problem D and Solution

The Standings



Problem

In a four team softball league, each team has played every other team 4 times. A team earned 3 points for a win, 1 point for a tie and no points for a loss. The total accumulated points were: Lions 22, Tigers 19, Mounties 14, and Royals 12. How many games ended in a win and how many games ended in a tie?

Solution

We begin by calculating the total number of games played. Since each team played every other team 4 times, each team played $3 \times 4 = 12$ games. Since there are four teams, a total of $\frac{4 \times 12}{2} = 24$ games were played. We divide by 2 since each game is counted twice. For example, the Lions playing the Tigers is the same as the Tigers playing the Lions.

In games where one team won and one team lost, one team earned 3 points and the other 0 points, so a total of 3 points were awarded. In games that resulted in a tie, both teams earned 1 point, so a total of 2 points were awarded.

If there were 0 ties, then 24 games would result in $24 \times 3 = 72$ points being awarded. However, $22 + 19 + 14 + 12 = 67$ points were actually awarded in all of the games. Since a total of 3 points were awarded when there was a win and a total of 2 points were awarded when there was a tie, every point below 72 must represent a tie. Since $72 - 67 = 5$, there must have been 5 ties. Since 24 games were played, $24 - 5 = 19$ games resulted in a win.

Therefore, there were 19 games that ended in a win and 5 games ended in a tie.

We should check that there is a combination of wins, ties and losses that satisfies the conditions in the problem. Indeed, one possibility is:

Team Name	Wins	Ties	Losses	Total Points
Lions	7	1	4	22
Tigers	6	1	5	19
Mounties	3	5	4	14
Royals	3	3	6	12
TOTALS	19	10	19	67

Notice, in the chart, that there are a total of 10 ties. That means that 5 games ended in a tie and a total of 10 points were awarded for ties.

Extension: There are 5 other combinations of wins, ties and losses that satisfy the conditions of the problem. Can you find them all?

