Problem of the Week
Problem D and Solution
From the Four Corners

Problem

In the diagram, \(ABCD\) is a rectangle. Point \(P\) is located inside the rectangle so that the distance from \(P\) to \(A\) is 5 cm, the distance from \(P\) to \(B\) is 11 cm, and the distance from \(P\) to \(D\) is 10 cm. How far is \(P\) from \(C\)?

Solution

We start by drawing a perpendicular from \(P\) to \(AB\). Let \(Q\) be the point of intersection. Let’s draw another perpendicular from \(P\) to \(DC\). Let \(R\) be the point of intersection.

Since \(QP\) is perpendicular to \(AB\), \(\angle AQP = 90^\circ\) and \(\angle BQP = 90^\circ\). Since \(PR\) is perpendicular to \(DC\), \(\angle DRP = 90^\circ\) and \(\angle CRP = 90^\circ\). We also have that \(AQ = DR\) and \(BQ = CR\).

We can apply the Pythagorean Theorem in \(\triangle AQP\) and \(\triangle BQP\).

From \(\triangle AQP\) we have \(AQ^2 + QP^2 = AP^2\), and so \(AQ^2 + QP^2 = 5^2 = 25\). Rearranging, we have \(QP^2 = 25 - AQ^2\) (1).

From \(\triangle BQP\) we have \(BQ^2 + QP^2 = BP^2\), and so \(BQ^2 + QP^2 = 11^2 = 121\). Rearranging, we have \(QP^2 = 121 - BQ^2\) (2).

Since \(QP^2 = QP^2\), from (1) and (2) we find that \(25 - AQ^2 = 121 - BQ^2\) or \(BQ^2 - AQ^2 = 96\). Since \(AQ = DR\) and \(BQ = CR\), this also tells us \(CR^2 - DR^2 = 96\) (3).

We can now apply the Pythagorean Theorem in \(\triangle DRP\) and \(\triangle CRP\).

From \(\triangle DRP\) we have \(DR^2 + RP^2 = DP^2\), and so \(DR^2 + RP^2 = 10^2 = 100\). Rearranging, we have \(RP^2 = 100 - DR^2\) (4).

When we apply the Pythagorean Theorem to \(\triangle CRP\) we have \(CR^2 + RP^2 = CP^2\). Rearranging, we have \(RP^2 = CP^2 - CR^2\) (5).

Since \(RP^2 = RP^2\), from (4) and (5) we find that \(100 - DR^2 = CP^2 - CR^2\), or \(CR^2 - DR^2 = CP^2 - 100\) (6).

From (3), we have \(CR^2 - DR^2 = 96\), so (6) becomes \(96 = CP^2 - 100\) or \(CP^2 = 196\). Thus \(CP = 14\), since \(CP > 0\).

Therefore the distance from \(P\) to \(C\) is 14 cm.