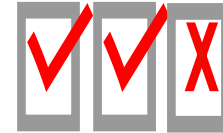




Problem of the Week

Problem D and Solution

Percents of Interest



Problem

The following information is known about the results of a recent math test: there were three questions on the test, each question was worth 1 mark, each question was marked right or wrong (no part marks), 50% of the students got all 3 questions correct, 5% of the students got no questions correct, and the class average mark was 2.3 out of 3. Determine the percentage of students who got exactly 1 question correct and the percentage of students who got exactly 2 questions correct.

Solution

Solution 1

In this solution, we will use only one variable.

To determine an average, we must determine the total of all the scores and divide by the number of students. Without changing the overall class average, suppose that 100 students wrote this test.

Let x represent the percent who got exactly 2 questions correct.

Then $100 - 50 - 5 - x = (45 - x)$ percent got exactly 1 question correct.

Since 50% of the students got all 3 questions correct, 50 students each scored 3 marks and earned a total of $50 \times 3 = 150$ marks.

Since $x\%$ of the students got exactly 2 questions correct, x students each scored 2 marks and earned a total of $x \times 2 = 2x$ marks.

Since $(45 - x)\%$ of the students got exactly 1 question correct, $(45 - x)$ students each scored 1 mark and earned a total of $(45 - x) \times 1 = (45 - x)$ marks.

Since 5% of the students got no questions correct, 5 students scored 0 marks and earned a total of $5 \times 0 = 0$ marks.

The total number of marks earned by the 100 students was $150 + 2x + (45 - x) + 0 = x + 195$.

We know that the average score was 2.3, so

$$\begin{aligned}\frac{x + 195}{100} &= 2.3 \\ x + 195 &= 230 \\ x &= 35 \\ 45 - x &= 10\end{aligned}$$

Therefore, 35% of the students got exactly 2 questions correct and 10% of the students got exactly 1 question correct.





Solution 2

In this solution, we will use two variables.

To determine an average, we must determine the total of all the scores and divide by the number of students. Without changing the overall class average, suppose that 100 students wrote this test.

Let x represent the percent who got exactly 2 questions correct.

Let y represent the percent who got exactly 1 question correct.

Then, $50 + x + y + 5 = 100$ which simplifies to $x + y = 45$. (1)

Since 50% of the students got all 3 questions correct, 50 students each scored 3 marks and earned a total of $50 \times 3 = 150$ marks.

Since $x\%$ of the students got exactly 2 questions correct, x students each scored 2 marks and earned a total of $x \times 2 = 2x$ marks.

Since $y\%$ of the students got exactly 1 question correct, y students each scored 1 mark and earned a total of $y \times 1 = y$ marks.

Since 5% of the students got no questions correct, 5 students scored 0 marks and earned a total of $5 \times 0 = 0$ marks.

The total number of marks earned by the 100 students was $150 + 2x + y + 0 = 2x + y + 150$.

We know that the average score was 2.3, so

$$\begin{aligned}\frac{2x + y + 150}{100} &= 2.3 \\ 2x + y + 150 &= 230 \\ 2x + y &= 80 \quad (2)\end{aligned}$$

Subtracting equation (1) from equation (2), we obtain $x = 35$. Substituting $x = 35$ into equation (1), we obtain $y = 10$.

Therefore, 35% of the students got exactly 2 questions correct and 10% of the students got exactly 1 question correct.





Solution 3

In this solution, we will use 2 variables but we will not assume a class size.

To determine an average, we must determine the total of all the scores and divide by the number of students.

Let n represent the number of students who wrote the test where n is a positive integer.

Let x represent the percent who got exactly 2 questions correct.

Let y represent the percent who got exactly 1 question correct.

Then, $50 + x + y + 5 = 100$ which simplifies to $x + y = 45$. (1)

Since 50% of the students got all 3 questions correct, $\frac{50}{100}n$ students each scored 3 marks and earned a total of $\frac{50}{100}n \times 3 = \frac{150n}{100}$ marks.

Since $x\%$ of the students got exactly 2 questions correct, $\frac{x}{100}n$ students each scored 2 marks and earned a total of $\frac{x}{100}n \times 2 = \frac{2xn}{100}$ marks.

Since $y\%$ of the students got exactly 1 question correct, $\frac{y}{100}n$ students each scored 1 mark and earned a total of $\frac{y}{100}n \times 1 = \frac{yn}{100}$ marks.

Since 5% of the students got no questions correct, 5 students scored 0 marks and earned a total of $5 \times 0 = 0$ marks.

The total number of marks earned by the n students was $\frac{150n}{100} + \frac{2xn}{100} + \frac{yn}{100} = \frac{n}{100}(2x + y + 150)$.

We know that the average score was 2.3 and n is a positive integer, so

$$\begin{aligned}\frac{\frac{n}{100}(2x + y + 150)}{n} &= 2.3 \\ 2x + y + 150 &= 230 \\ 2x + y &= 80 \quad (2)\end{aligned}$$

Subtracting equation (1) from equation (2), we obtain $x = 35$. Substituting $x = 35$ into equation (1), we obtain $y = 10$.

Therefore, 35% of the students got exactly 2 questions correct and 10% of the students got exactly 1 question correct.

