Problem of the Week
Exponential Primer

Problem
We are given two expressions:

<table>
<thead>
<tr>
<th>Expression A</th>
<th>Expression B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$72 \times 5^x$</td>
<td>$225 \times 2^y$</td>
</tr>
</tbody>
</table>

Given that $x$ and $y$ are positive integers, find all ordered pairs $(x, y)$ so that Expression $A$ equals Expression $B$.

Solution

Solution 1
We write each expression as the product of prime factors.
Expression $A = (2^3)(3^2)(5^x)$ and Expression $B = (3^2)(5^2)(2^y)$.
Since $x$ and $y$ are each positive integers and the expressions are equal, then the corresponding exponents for each prime number must be equal. Therefore, $x = 2$ and $y = 3$ are the only integer solutions for $x$ and $y$, and the only ordered pair is $(2, 3)$.

Solution 2
Setting the two expressions equal to each other,

$$72 \times 5^x = 225 \times 2^y.$$ 

Dividing both sides by 9,

$$8 \times 5^x = 25 \times 2^y.$$ 

Expressing each side of the equation as the product of prime factors,

$$2^3 \times 5^x = 5^2 \times 2^y.$$ 

Since $x$ and $y$ are each positive integers and the expressions are equal, then the corresponding exponents for each prime number must be equal. Therefore, $x = 2$ and $y = 3$ are the only integer solutions for $x$ and $y$, and the only ordered pair is $(2, 3)$. 