Problem of the Week
Problem C and Solution
One Off

Problem

Gregor has decided to create a new sequence that starts with the numbers $4, 1, 5, 6, 11, \ldots$ where each term after the first two terms is found by adding the two previous terms. On the 25th term, his addition was off by 1. That is, his 25th term was 1 greater than the correct one. If he made no other mistakes, how far off was the 35th term?

$$\text{Term } 1 = 4$$
$$\text{Term } 2 = 1$$
$$\text{Term } 3 = \text{Term } 1 + \text{Term } 2 = 4 + 1 = 5$$
$$\text{Term } 4 = \text{Term } 2 + \text{Term } 3 = 1 + 5 = 6$$
$$\text{Term } 5 = \text{Term } 3 + \text{Term } 4 = 5 + 6 = 11$$

Solution

Solution 1

We could write out the whole sequence and then the incorrect sequence.

Here are the first 24 correct numbers:

4, 1, 5, 6, 11, 17, 28, 45, 73, 118, 191, 309, 500, 809, 1309, 2118, 3427, 5545, 8972, 14 517, 23 489, 38 006, 61 495, 99 501

Here are the correct numbers from term 25 to term 35:

160 996, 260 497, 421 493, 681 990, 1 103 483, 1 785 473, 2 888 956, 4 674 429, 7 563 385, 12 237 814, 19 801 199

And here are the incorrect terms after the 25th term is increased by 1:

160 997, 260 498, 421 495, 681 993, 1 103 488, 1 785 481, 2 888 969, 4 674 450, 7 563 419, 12 237 869, 19 801 288

The difference between the correct 35th term and the incorrect 35th term is $19\,801\,288 - 19\,801\,199 = 89$. Therefore, the 35th term is off by 89.

Solution 2

The 25th term is off by one.

Therefore, the 26th term will also be off by 1 since it equals the sum of the 24th term (which is unchanged) and the 25th term (which is off by 1). This also means the 27th term will be off by 2 since it is the sum of the 25th term (which is off by 1) and the 26th term (which is off by 1). Furthermore, the 28th term is off by 3 since it is the sum of the 26th term (which is off by 1) and the 27th term (which is off by 2). This pattern will continue on.

Let's summarize this in a table.

Term Number N	Amount Above the Original AA
24	0
25	1
26	1
27	2
28	3
29	5
30	8
31	13
32	21
33	34
34	55
35	89

Therefore, the 35th term is off by 89.

Notice that the terms in column AA follow the same rule as the original question. That is, to find the Amount Above the Original after term 25, add the previous 2 terms.

A FURTHER NOTE: The last 11 numbers in column AA are the first 11 numbers of a famous sequence known as the Fibonacci Sequence. You may wish to investigate the Fibonacci Sequence further.