Problem of the Week Problem C and Solution Odd or Even?

Problem

In a sequence of numbers, each number in the sequence is called a term. In the sequence 1, 3, 5, 7, 9, the first term is 1, the second term is 3, the third term is 5, the fourth term is 7 and the fifth term is 9.

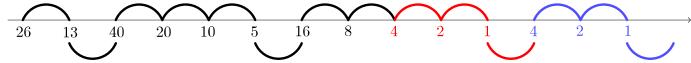
In another sequence, the first term is 26. If a term in the sequence is even, then the next term will be half the value of that term. If a term in the sequence is odd, then the next term in the sequence is one more than three times that term. By following these rules, the first three terms of this sequence are 26, 13 and 40.

Is the twenty-sixth term of the sequence odd or even? Justify your answer.

Solution

We will begin by finding more terms in the sequence.

To get from an even term to the next term, divide the term by 2



To get from an odd term to the next term, multiply the term by 3 and then add 1

The first 14 terms of the sequence are 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1.

Terms 9, 10 and 11 of the sequence are 4, 2 and 1, respectively.

Terms 12, 13 and 14 of the sequence are 4, 2 and 1, respectively.

The three numbers 4, 2 and 1 will continue to repeat.

So the 9th term, 12th term, 15th term, and so on will each have a value of 4. The n^{th} term of the sequence will equal 4 for integer values of $n \geq 9$ that are also multiples of 3. Therefore, the 24th term will also be 4.

It then follows that the 25th term will be 2 and the 26th term will be 1. The 26th term is an odd number.

