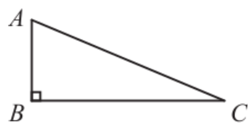
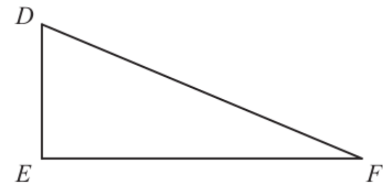


Problem of the Week

Problem C and Solution

Scaling Up

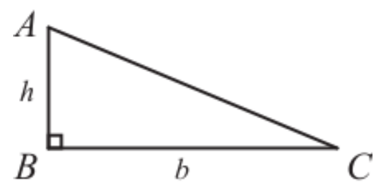


Problem

A student constructs a right-angled triangle, $\triangle ABC$, with an area of 6 cm^2. She constructs a second triangle, $\triangle DEF$, whose side lengths are exactly three times the lengths of the sides of her original triangle. That is, $DE = 3AB$, $EF = 3BC$ and $DF = 3AC$. Given this information, determine the area of $\triangle DEF$.

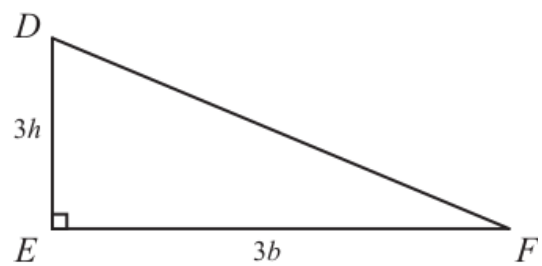
Solution

Let h represent the height of $\triangle ABC$ and b represent the base of $\triangle ABC$.



Using the formula area $= \frac{\text{base} \times \text{height}}{2}$, we have $6 = \frac{b \times h}{2}$, or $b \times h = 12$.

$\triangle DEF$ is formed by multiplying each of the original side lengths by 3. Since the ratio of the side lengths does not change, that is, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{1}{3}$, then $\triangle ABC \sim \triangle DEF$.



Since corresponding angles in similar triangles are equal, then $\angle DEF = \angle ABC = 90^\circ$, and so $\triangle DEF$ is also right-angled.
Since $\triangle ABC$ has height h, then $\triangle DEF$ has height $3 \times h$.
Since $\triangle ABC$ has base b, then $\triangle DEF$ has base $3 \times b$.

Using the formula area $= \frac{\text{base} \times \text{height}}{2}$,

$$\begin{aligned} \text{the area of } \triangle DEF &= \frac{(3 \times b) \times (3 \times h)}{2} \\ &= \frac{9 \times b \times h}{2} \\ &= \frac{9 \times 12}{2}, \text{ since } b \times h = 12 \\ &= 54 \text{ cm}^2 \end{aligned}$$

As an extension:

Notice that $\triangle DEF$ has side lengths that are each three times the corresponding lengths of $\triangle ABC$ and that the area of the $\triangle DEF$ ended up being $54 = 9 \times 6 = 3^2 \times$ area of $\triangle ABC$.

This is not a coincidence. It turns out that if $\triangle ABC \sim \triangle DEF$ and the side lengths of $\triangle DEF$ are all k times the corresponding side lengths of $\triangle ABC$, then

$$\text{area of } \triangle DEF = k^2 \times \text{area of } \triangle ABC$$

Can you show that this is always true for a right-angled triangle? Can you show that this is always true for **any** triangle?