Problem of the Week
Problem C and Solution
Under the Big Top

Problem
A large tent is being set up for a fair. Two poles, $QU$ and $RT$, are placed perpendicular to the ground and 12 m apart. Pole $QU$ is 4 m in length and pole $RT$ is 7.5 m in length. A tarp is placed over the poles and secured to the ground at $P$, 3 m from the base of pole $QU$, and $S$, 4 m from the base of pole $RT$.

Determine $PQ + QR + RS$, the length of the tarp.

Solution
First, we will calculate $PQ$ and $RS$.

Since $\angle PUQ = 90^\circ$, we can apply the Pythagorean Theorem in $\triangle PUQ$. Thus $PQ^2 = QU^2 + PU^2 = 4^2 + 3^2 = 16 + 9 = 25$. Therefore, $PQ = 5$, since $PQ > 0$.

Similarly, $\angle RTS = 90^\circ$, so we can apply the Pythagorean Theorem in $\triangle RTS$. Thus $RS^2 = RT^2 + TS^2 = 7.5^2 + 4^2 = 56.25 + 16 = 72.25$. Therefore, $RS = 8.5$, since $RS > 0$.

Now we will calculate $QR$.

Draw a line from $Q$ perpendicular to $RT$.

Let $A$ be the point of intersection of the perpendicular with $RT$. Since $QA$ is perpendicular to $RT$, $QATU$ is a rectangle.

Therefore, $QA = UT = 12$ and $AT = QU = 4$.

Thus $AR = RT - AT = 7.5 - 4 = 3.5$.

Since $\angle QAR = 90^\circ$, we can apply the Pythagorean Theorem in $\triangle QAR$. Thus $QR^2 = QA^2 + AR^2 = 12^2 + 3.5^2 = 144 + 12.25 = 156.25$. Therefore, $QR = 12.5$, since $QR > 0$.

Therefore, the length of the tarp is $PQ + QR + RS = 5 + 12.5 + 8.5 = 26$ m.